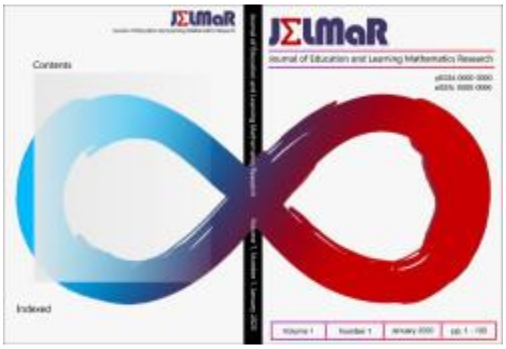

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Forward Time Center Space Algorithm for Mathematical Model Solution of Heat Transfer

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***Abstract**: Heat transfer is a physical phenomenon that can be represented in the form of a mathematical model. In this study, heat transfer occurs in a viscoelastic fluid through an elliptic cylinder surface with free convection flow. The mathematical model of heat transfer is obtained from partial differential equations and solved numerically using the Forward Time Center Space (FTCS) scheme. Numerical solution is carried out based on an algorithm compiled by an iterative process according to a predetermined point. The iteration process is carried out until it produces a stable and convergent value. Furthermore, the algorithm is implemented into the Matlab programming language with the influence of a heat variable, namely the Prandtl number (Pr). Several test results that have been carried out during the iteration process have shown that the FTCS scheme is stable along the space and time grid. In addition, this scheme shows that the obtained difference equations are proven to produce consistent and convergent graphs. Based on the resulting graph, the greater the value of the Prandtl number, the smaller the resulting temperature. This is in accordance with the definition of the Prandtl number, which is the heat determining parameter which is the ratio between the kinematic viscosity value and the heat diffusivity, so that the large Prandtl number can inhibit heat transfer that occurs on the surface of the object.*

Keyword: Algorithm, Forward Time Center Space, Mathematical Model, Heat Transfer

INTRODUCTION

Heat transfer is a physical phenomenon that can be represented in the form of a mathematical model. The mathematical model for heat transfer is obtained from the differential equation. Differential equations are studies in the field of mathematics that are widely used to solve problems in the fields of physics, chemistry, economics, industry, engineering, and other scientific disciplines that can describe complex natural phenomena. (Havid Syafwan, Mahdhivan Syafwan, William Ramdhan, 2018).

To solve the differential equation numerically, it is necessary to first find the approximation of the derived terms in the differential equation. One of the methods used to calculate the approximate derivative of a function is the finite difference method. Based on the location of the partition point used, the finite difference method is divided into three types, namely forward difference, backward difference, dan central difference.

This research focuses on the analysis of heat transfer on the surface of an elliptical cylinder in an unsteady condition and the flow is incompressible. The numerical method used is the finite difference method with the Forward Time Center Space (FTCS) scheme as part of the explicit finite difference scheme. The numerical solution is based on an algorithm compiled by an iterative process according to a predetermined point. Explicit formulas are designed using the Matlab programming language, so that it can make it easier to calculate numerical derivatives of functions.

Research on finite difference numerical solutions for heat transfer has also been carried out before, including (Afifah & Putra, 2018; Cheng, 2012; El Maghfiroh et al., 2019; Hapsoro & Srigutomo, 2018; Imron et al., 2013; Kasim, 2014; Mahat et al., 2017; Mardianto, 2018; Martanegara & Yulianti, 2020; Mohammad, 2014; Pendahuluan, 2019; Purnami et al., 2018; Sahaya et al., 2016; Tiwow et al., 2015).

METHOD

The finite difference method with the Forward Time Center Space Scheme is used to calculate the approximate derivative of a function. In this scheme the domain of the function f(x) is partitioned into a number of points and an approximation formula for the derivative obtained from the Taylor series expansion. The steps for implementing the research are as follows:

1. Discretize the mathematical model using the FTCS Scheme;
2. Designing data structures;
3. Arrange algorithm;
4. Translating algorithms into programming language code;
5. Arrange the code into a computer program;
6. Running computer programs;
7. Analyze the results of the visualization.

In simple terms, the research flow is described in the form of the following Program Flowchart.

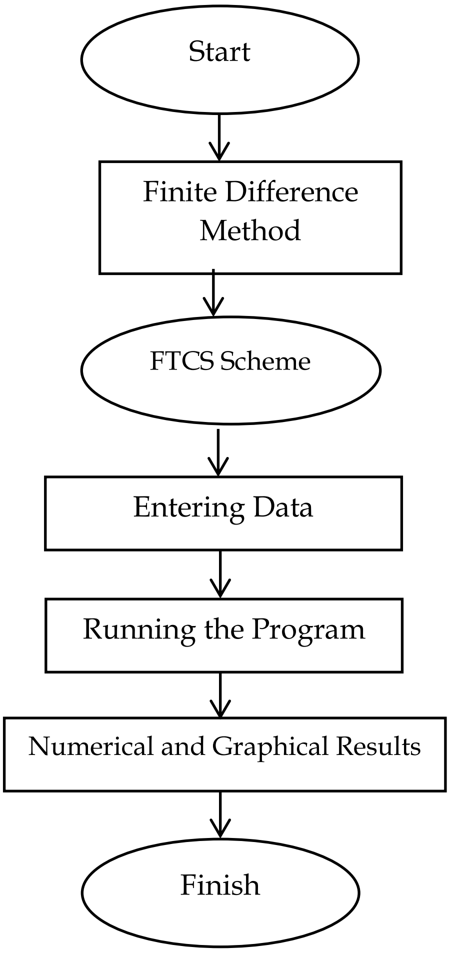

Figure 1. Program Flowchart

RESULT AND DISCUSSION

Discretization of Mathematical Models

The mathematical model of heat transfer for viscoelastic fluid flows under unsteady conditions and incompressible flow properties is as follows (Annisa dwi sulistyanigsih, 2021):

$$f''' + ff'' - (f')^2 + \theta sinA - K(2f'f''' - ff^{(4)} - (f'')^2 = 0 \quad (1)$$

$$\frac{1}{Pr}\theta'' + f\theta' + \gamma\theta = 0 \quad (2)$$

with boundary conditions

$$f(0) = f'(0) = 0, \quad \theta'(0) = -1$$
$$f'(\infty) = 0, \; f''(\infty) = 0, \quad \theta(\infty) = 0$$

Next, numerical processing is carried out for Equations (1) and (2) based on the following FTCS Method scheme.

$$f' = p$$
$$f = p\Delta y$$

So that Equations (1) and (2) can be expressed in the form:

$$p'' + p\Delta yp' - p^2 + \theta sinA - K(2pp'' - p\Delta yp''' - (p')^2) = 0 \quad (3)$$

$$\frac{1}{Pr}\theta'' + f\theta' + \gamma\theta = 0 \quad (4)$$

Based on Figure 2, the application of the center difference to p at points i,j is in the form of the following derivative scheme.

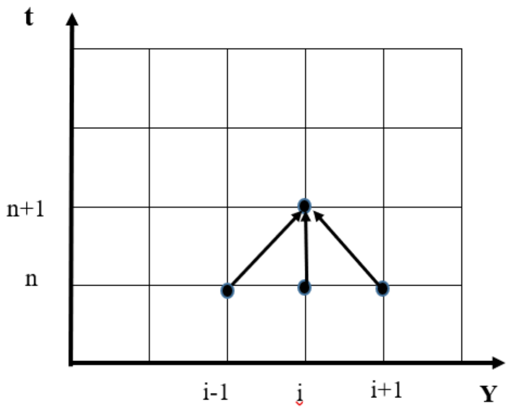

Figure2. FTCS Scheme

Schema for the first derivative:

$$\left.\frac{\partial p}{\partial y}\right|_{y_i} = \frac{p_{i+1} - p_{i-1}}{2\Delta y} + O(\Delta y^2)$$

Schema for the second derivative:

$$\left.\frac{\partial^2 p}{\partial y^2}\right|_{y_i} = \frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta y^2} + O(\Delta y^2)$$

Schema for the third derivative:

$$\left.\frac{\partial^3 p}{\partial y^3}\right|_{y_i} = \frac{p_{i+2} - 2p_{i+1} + 2p_{i-1} - p_{i-2}}{2\Delta y^3} + O(\Delta y^2)$$

By using the derivative function scheme, Equation (3) can be stated as follows.

$$\frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta y^2} + p_i \Delta y \frac{p_{i+1} - p_{i-1}}{2\Delta y} - p_i^2 + \theta_i sinA - K\left(2p_i \frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta y^2} - p_i \Delta y \frac{p_{i+2} - 2p_{i+1} + 2p_{i-1} - p_{i-2}}{2\Delta y^3} - \left(\frac{p_{i+1} - p_{i-1}}{2\Delta y}\right)^2\right) = 0 \qquad (5)$$

For equation (4), discretization is carried out in the same way, resulting in the following equation.

$$\frac{1}{Pr}\left(\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{\Delta y^2}\right) + f_i\left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta y}\right) + \gamma\theta_i = 0$$

by assuming

$$r_1 = \frac{1}{Pr\Delta y^2}\ , \quad r_2 = \frac{1}{2\Delta y}$$

then we get the equation

$$\theta_i = \frac{(r_1 + r_2 f_i)\theta_{i+1} + (r_1 - r_2 f_i)\theta_{i-1} + 2\Delta y}{(2r_1 - \gamma)} \qquad (6)$$

Iteration Process

At this stage, an iteration process is carried out for Equation (5). By assuming,

$$A = \frac{1}{\Delta y^2},\ B = \frac{1}{2} - \frac{5}{2}KA, \qquad C = -\frac{1}{2} - \frac{3}{2}KA, \qquad D = KA, \qquad E = 1 - 4KA$$

Then the iteration process is then carried out using for i=1,2,3,…,M with predetermined boundary conditions, the following equations are obtained:

When i=1

$$p_i = \left(\frac{\left(Ap_{i+1} + (-2A)p_i + Bp_i p_{i+1} + \theta_i sinA + \frac{1}{2}Dp_i p_{i+2} - \frac{1}{2}Dp_i p_i + \frac{1}{4}Dp_{i-1}^2\right)}{E}\right)^{\frac{1}{2}}$$

When i=2

$$p_i = \left(\frac{\left(\begin{array}{c} Ap_{i+1} + (-2A)p_i + Ap_{i-1} + Bp_i p_{i+1} + Cp_i p_{i-1} + \theta_i sinA + \frac{1}{2}Dp_i p_{i+2} + \\ \frac{1}{4}Dp_{i+1}^2 - \frac{1}{2}Dp_{i+1}p_{i-1} + \frac{1}{4}Dp_{i-1}^2 \end{array}\right)}{E}\right)^{\frac{1}{2}}$$

When i=M-1

$$p_i = \left(\frac{\left(\begin{array}{c} Ap_{i+1} + (-2A)p_i + Ap_{i-1} + Bp_ip_{i+1} + Cp_ip_{i-1} + \theta_i sinA + \frac{1}{2}Dp_ip_i + \\ \frac{1}{4}Dp_{i+1}^2 - \frac{1}{2}Dp_{i+1}p_{i-2} - \frac{1}{2}Dp_{i+1}p_{i-1} + \frac{1}{4}Dp_{i-1}^2 \end{array} \right)}{E} \right)^{\frac{1}{2}}$$

When i=M

$$p_i = \left(\frac{\left(\begin{array}{c} Ap_{i+1} + (-2A)p_i + Ap_{i-1} + Bp_ip_{i+1} + Cp_ip_{i-1} + \theta_i sinA + \frac{1}{2}Dp_ip_{i+2} + \\ \frac{1}{4}Dp_{i+1}^2 - \frac{1}{2}Dp_{i+1}p_{i-2} - \frac{1}{2}Dp_{i+1}p_{i-1} + \frac{1}{4}Dp_{i-1}^2 \end{array} \right)}{E} \right)^{\frac{1}{2}}$$

As for equation (6), we get
When $i = 1$

Change θ_{i-1} with $\theta_{i+1} + 2\Delta y$, so that it is obtained

$$\theta_i = \frac{(r_1 + r_2 f_i)\theta_{i+1} + (r_1 - r_2 f_i)\theta_{i+1} + 2\Delta y}{(2r_1 - \gamma)}$$

When $i = M$

Change θ_{i+1} with 0, so that it is obtained

$$\theta_i = \frac{(r_1 - r_2 f_i)\theta_{i-1}}{(2r_1 - \gamma)}$$

Algorithm Creation

This stage describes the algorithm that is compiled based on the results of the discretization and the iteration process that has been carried out. The algorithm is structured in the following programming language.

Algorithm 1 Algoritma Penyelesaian Numerik dengan Menggunakan Matlab

Set Input: nilai variasi parameter, ketebalan lapisan batas (Δy), panjang ketebalan lapisan batas (y).

procedure
Kondisi awal $p(i), f(i)$, dan $\theta(i)$
for $it_{max} = 1$ sampai 5000 **do**
for $i = 1$ sampai M **do**

$$p_i = \left(\left(\mathrm{A}p_{i+1} + (-2\mathrm{A})p_i + \mathrm{A}p_{i-1} + \mathrm{B}p_ip_{i+1} + \mathrm{C}p_ip_{i-1} + \theta_i \sin A + \frac{1}{2}\mathrm{D}p_ip_{i+2} - \frac{1}{2}\mathrm{D}p_ip_{i-2} + \frac{1}{4}\mathrm{D}p_{i+1}^2 - \frac{1}{2}\mathrm{D}p_{i+1}p_{i-1} + \frac{1}{4}\mathrm{D}p_{i-1}^2\right)/\mathrm{E}\right)^{\frac{1}{2}}$$

$$\theta_i = ((r_1 + r_2 f_i)\theta_{i+1} + (r_1 - r_2 f_i)\theta_{i-1})/((2r_1 - \gamma)$$

if $i = 1$ **then**
$p_{i-1} \leftarrow 0$
$p_{i-2} \leftarrow p_i$
$\theta_{i-1} \leftarrow \theta_{i+1} + 2\Delta y$
if $i = 2$ **then**
$p_{i-2} \leftarrow 0$
if $i = M - 1$ **then**
$p_{i+2} \leftarrow p_i$
if $i = M$ **then**
$p_{i+1} \leftarrow p_{i-1}$
$p_{i+2} \leftarrow p_i$
$\theta_{i+1} \leftarrow 0$
end if
end if
end if
end if
end for
end for
end procedure

Figure 3 Heat Transfer Algorithm with FTCS Skema Scheme

Simulation Results

Based on the algorithm in Figure 3, the temperature profile graph is obtained as follows.

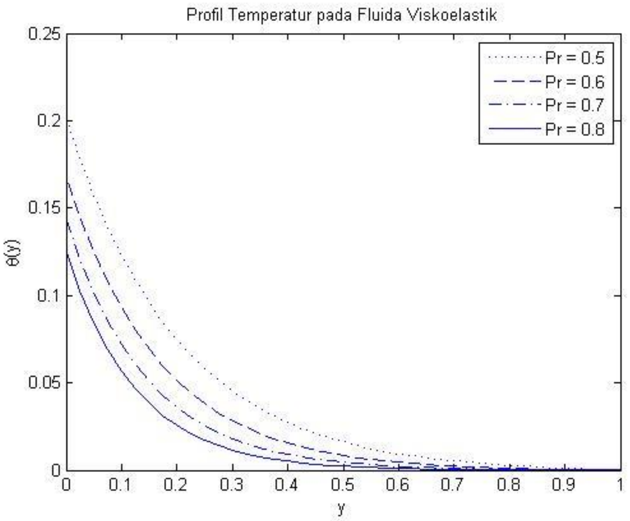

Figure 4 Heat Transfer Profile on Elliptical Cylinder Surface

Figure 4 is the visualization result of the algorithm that has been compiled with variations of Prandtl Numbers (Pr), namely 0.5, 0.6, 0.7, 0.8 and uses the value $\gamma = 1, K = 0.5, a = 10, b = 5$.. The results obtained in the form of a heat transfer profile on the surface of an elliptical cylinder with a heat variable, namely the Prandtl Number (Pr). The resulting graph is a consistent and convergent graph. The greater the value of the Prandtl number, the smaller the resulting temperature. This is in accordance with the definition of the Prandtl number, which is the heat determining parameter which is the ratio between the kinematic viscosity value and the heat diffusivity, so that the large Prandtl number can inhibit heat transfer that occurs on the surface of the object.

CONCLUSION

Based on the results and discussion above, it can be concluded that the heat transfer of a fluid on the surface of an elliptical cylindrical object can be represented in the form of a mathematical model. The mathematical model that is built from an unsteady and incompressible flow can be simulated into an algorithm, resulting in a graph of a function in the form of a heat transfer profile that is solved numerically using the Forward Time Center Space (FTCS) scheme. Furthermore, the algorithm is translated using a programming language and visualized with Matlab software. When running the computer program, an iteration process is carried out so as to produce a graph of a stable and convergent heat transfer profile. From the graph, it can be seen that the variation of the Prandtl Number affects the heat transfer that occurs on the surface of the object. The larger the given Prandtl number, the slower the heat distribution that occurs. This is because the value of the kinematic viscosity that is generated is getting bigger which causes the frictional force between the fluid and the surface of the object to be large.

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