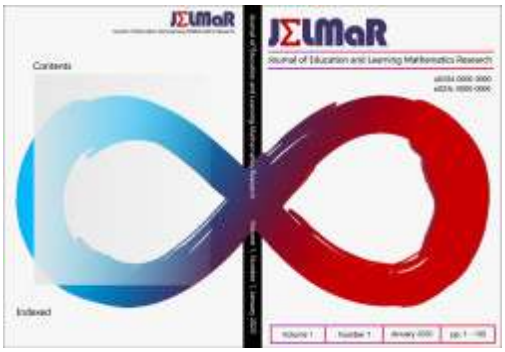

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Analyzing Students' Understanding of the Concepts of Congruence and Resemblance through the Design of Folklore HLTs

[1]Maftuhah, [2*]Eric Dwi Putra, [3]Nur Hidayatin
Mathematics Education Study Program, Faculty of Teacher Training and Education, PGRI Argopuro University of Jember
[*]*Email: dwieric454@gmail.com*

***Abstract**: This study aims to analyze students' understanding of the concept of congruence and congruence through the design of Hypothetical Learning Trajectory (HLT) based on folklore. The research subjects were students of grade VIII at Ahlul Irfan Junior High School. Data were collected through observation, interviews, comprehension tests, and documentation during the learning process. Data analysis was conducted using data reduction, data presentation, and conclusion drawing techniques. The results showed that students with high ability were able to meet all indicators of understanding, such as restating concepts, categorizing objects, giving examples and non- examples, using various mathematical representations, choosing the right procedures, and applying concepts in problem solving. In contrast, students with low ability were only able to fulfill two indicators, namely restating concepts and categorizing objects based on certain characteristics, while they were not able to use representations, procedures, and logical reasoning in problem solving. This study reinforces the importance of using local cultural contexts in mathematics learning to address misconceptions and increase student engagement. Teachers are recommended to use folklore-based HLT as a contextual and interesting learning media, with special attention to assisting low ability students.*

***Keyword**: Understanding, Congruence, Equivalence, HLT, Folklore*

INTRODUCTION

Comprehension is an important cognitive aspect in the learning process. Radiusman (2020) explains that understanding is related to students' ability to capture meaning and master a problem thoroughly. Meanwhile, Mawaddah & Maryanti (2016) emphasized that the ability to explain and interpret concepts, provide illustrations, more complete explanations, and produce more innovative explanations are all part of the comprehension process. In improving knowledge, a strong understanding of concepts will make it easier for students to understand what they are learning (Khoerunnisa & Sari, 2021). One of the subjects that really requires a strong level of concept understanding is mathematics.

Mathematics is a very important component for the success of the education program (Putra & Lutfiyah, 2020). However, not a few students consider mathematics as a difficult subject because its abstract nature involves patterning, and demands the ability to think systematically, critically, logically, and consistently (Masjudin, 2016). Rahayu (2016) said in her article that students often make mistakes when solving math problems, especially complex ones. The most common errors are concept, principle, and operation errors. Mathematics material that is considered difficult by ninth grade students is the material of congruence and perpendicularity.

Research conducted by Fadilah & Bernard (2021) found that 51.4% of students still make mistakes in understanding a problem related to congruence and similarity material. Safitri (2020) said that students often cannot distinguish that two similarity flat shapes are not necessarily congruent, and that two congruent flat shapes are definitely similarity. In her research Nurfajriah (2022) said learners may experience this difficulty because they are

less careful in understanding the problem, which makes it difficult to find the right concept. The right strategy is needed to help students solve the problem correctly (Putra & Sari, 2023). Cultural and social values that develop in the community will help the process of understanding mathematics at school (Nova & Putra, 2022). Based on this, this study utilizes folklore visualized in the form of images as a learning context to design activities to facilitate students' understanding of the concepts of congruence and similarity.

This research uses the folklore "Asal Usul Watu Ulo" as a learning context. This story was chosen because it contains visual elements that can be associated with the concepts of congruence and similarity, and is close to the lives of students in the area of origin of the story. Relevant research was conducted by Sary (2017) which focused on designing HLT by utilizing folklore as a context for learning mathematics, especially on comparative value material. The results of her research showed that the use of local cultural contexts could help students understand the material more meaningfully. However, research using folklore as a visual context to design HLT on the concept of congruence and congruence is still rare. Therefore, this research was conducted as an effort to fill the gap. This research uses the Hypothetical Learning Trajectory (HLT) approach to design learning for the concepts of congruence and congruence with the context of the folklore "Asal Usul Watu Ulo". The Hypothetical Learning Trajectory (HLT) approach is used as a framework in designing learning. According to Czarnocha (2016, in Putri, 2012), HLT consists of three main parts, namely: (a) learning objectives, (b) learning activities designed to achieve those objectives, and (c) hypotheses about students' thinking or learning processes while following those activities. This framework allows teachers or researchers to predict and direct the course of learning systematically.

Based on the theoretical review and existing problems, this research aims to analyze students' understanding of the concepts of congruence and similarity through the design of Hypothetical Learning Trajectory (HLT) developed with the context of folklore "Asal Usul Watu Ulo".

METHOD

This study used a descriptive qualitative approach with a research design research type. This descriptive qualitative approach was chosen to describe and understand the phenomenon in depth through narratives or words (Waruwu, 2023). While the type of design research was chosen because it is in accordance with the research objectives to design, implement, and analyze the learning process on the concept of congruence and similarity through the design of Hypothetical Learning Trajectory (HLT) based on folklore. The focus of this research is to find out how learning using folklore can contribute to students' understanding of the concepts of congruence and similarity. The research was conducted at Ahlul Irfan Junior High School, located in Tisnogambar Village, Bangsalsari District, Jember Regency. The subjects in this study were ninth grade students, because the material of congruence and perpendicularity is taught at that level. The selection of subjects was carried out using purposive sampling technique, namely choosing intentionally based on certain criteria (Lenaini, 2021). A total of four students were selected as research subjects, consisting of two high ability students and two low ability students, based on the results of interviews with mathematics teachers. Data collection techniques in this study include observation, interviews, written tests, and documentation. Data analysis was carried out following the stages according to Rijali (2018), namely data reduction, data presentation, and conclusion drawing.

Table 1. Indicators of understanding of mathematical concepts

No	Indicators of understanding	Student activity
1	Restate a concept	Students use their own words to show what they understand.
2	Categorize objects based on certain characteristics shared by the concept	Students are given several pairs of flat shapes and asked to categorize them based on congruence and similarity.
3	Giving examples and non-examples of concepts	Students are able to demonstrate their understanding of the difference between congruence and similarity.
4	Display concepts in various forms of mathematical representations	Students explain in words how two shapes are congruent or similar.
5	Use, utilize, and select certain procedures or operations	Students use math formulas that they have they have learned to solve problems.
6	Apply concepts or theorems to solve problem	Students use concepts or theorems that they have learned to solve math problems.

The HLT in this study was developed based on three main components, namely learning objectives, learning activities, and learning hypotheses (Czarnocha, 2016). Details of these three components are shown in Table 2.

Table 2. HLT design for the folklore "watu ulo"

Learning objectives	Learning activity	Learning hypothesis
Students can recognize congruent and similar shapes from images of Watu Ulo stones.	Presenting a picture of Watu Ulo stones, asking students to observe shapes that look similar.	Students will identify similar shapes as "the same," but may not yet be able to distinguish between congruent and similar shapes. Teachers will explore students' answers in greater depth and ask follow-up questions.
Students can compare sizes and angles to determine congruence or similarity.	Assign students to measure the side lengths and angle magnitudes of two rock drawings.	Some students will notice differences in size and angle, but will not yet be able to conclude which are congruent and which are similar. Teachers help students clarify definitions with concrete examples.
Students can formulate their own definitions of congruence and similarity.	Group discussion to develop definitions based on observations and measurements.	Students will propose definitions that are close to formal concepts. The teacher will lead to the correct definition through questions and answers.

RESULT AND DISCUSSION

This chapter presents the results of the implementation and analysis of the Hypothetical Learning Trajectory (HLT) design based on the folklore of Asal Usul Watu Ulo in learning the concepts of congruence and similarity. The main focus of this chapter is to reveal how students' understanding of the two concepts develops through the activities designed in the HLT. The results obtained were analyzed to see the extent to which students

understood the concepts of congruence and similarity, and how the folklore used was able to facilitate the understanding process.

HLT implementation in the classroom

Activity 1 (observing patterns)

In Activity 1, students are divided into small groups and given a Group Worksheet containing a modified picture of Watu Ulo. The picture has been modified by inserting various triangular flat shapes arranged to resemble the pattern of the scales or body of Watu Ulo with variations in shape and size.

The teacher directs students to observe the patterns of flat shapes in the picture. After that, students are asked to identify which flat shapes have the same shape and size (congruent), and which shapes have the same shape but different sizes (similar).

Figure 1. Illustration of the modified watu ulo

Figure 2. Students discussing grouping flat shapes

Most students are able to visually identify congruent shapes, especially if they are parallel and close to each other. When faced with shapes that have the same shape but different sizes, some students state that it is "the same", so the teacher asks further to explore their reasoning.

Teacher : "Look at the scales of Watu Ulo, which ones have the same shape and the same size?"
Student A: "This one and this one, Mom! The two triangles here are the right size."
Teacher : "If this triangle is bigger than the other one, can it still be called equal?"
Student B : "The shape is the same, but the size is different."

This activity is effective as an introduction to the concept of congruent and similarity, as it involves interesting visuals and local context. However, students' understanding of the difference between same shape and same size still needs to be strengthened in the next activity.

Activity 2. (proving congruence and similarity)

In this activity, students are given worksheets containing pictures of flat shapes found on the body of Watu Ulo. The picture has been modified by adding special marks on the sides of the flat shapes such as small lines or parallel line symbols to show that the sides are the same length. Students are asked to Observe the signs on the sides of the flat shape, Measure the length of the sides, and Record the measurement results into the table on the Worksheet, which consists of: Name of the shape, Length of the sides, Size of the angles, Conclusion: congruent or same shape but different size (similar).

KEGIATAN 2 (Membuktikan kekongruenan dan kesebangunan)

Ukurlah panjang sisi-sisi pada bangun yang kamu temukan tadi. (Tuliskan hasil pengukuranmu!)

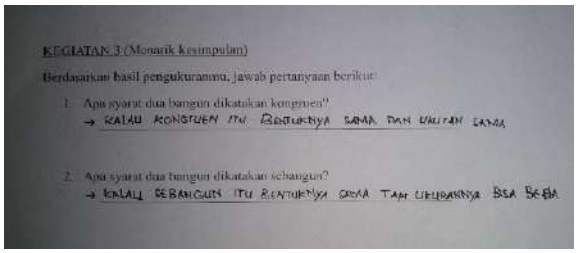
Bangun	Sisi 1 (cm)	Sisi 2 (cm)	Sisi 3 (cm)	Sudut (apakah sama?)	Keterangan
AABC dan ABCD	Tandanya samasisi sama Panjang)	Tandanya sama	Tandanya sama		AABC dan ABCD merupakan kongruen karena setiap sisinya terdapat tanda yang menunjukkan sama panjang
ΔMNO dan ΔNPQ	...				
ΔPQR dan ΔSQT	...				

Figure 3. Group worksheet for activity 2 - Proving congruence and similarity

Symbols or marks on the image greatly help students predict similarities between sides before taking measurements. This stimulates the connection between visual observation and actual measurement, which is important for building a complete understanding of the concept of congruence. However, there are still students who are unable to accurately distinguish between visual similarities and similarities in size. Some students tend to consider shapes congruent merely because they look similar or because there are marks on the sides, even though measurement results show different sizes. This highlights the need for further guidance to strengthen students' understanding of the difference between congruence and similarity.

Activity 3 (drawing conclusions)

After students observe the shapes of flat shapes (activity 1) and compare sizes and shapes through filling in the table (activity 2), in this third activity students are directed to conclude the difference between congruent and similarity shapes.



KEGIATAN 3 (Menarik kesimpulan)

Berdasarkan hasil pengukuranmu, jawab pertanyaan berikut!

1. Apa syarat dua bangun dikatakan kongruen?
   → KALAU KONGRUEN ITU BENTUKNYA SAMA DAN UKURAN SAMA
2. Apa syarat dua bangun dikatakan sebangun?
   → KALAU SEBANGUN ITU BENTUKNYA SAMA TAPI UKURANNYA BISA BEDA

Figure 4. Group 2 Worksheet in activity 3 - Drawing Conclusions about congruence and similarity

Group 2's Worksheet shows the results of their discussion in drawing conclusions about the concepts of congruence and similarity. At the end of the worksheet, the group wrote that congruent shapes have the same shape and size, while congruent shapes have the same shape but different sizes. Although there are still a few mistakes in the use of formal terms and in terms of letters, the answers written show that students have understood the essence of the difference between the two concepts based on observing and measuring activities in the previous activity. This picture also shows how students write the results of group discussions independently in their own language, which reflects the process of internalizing concepts through contextual and visual-based activities.

Description and analysis of students' understanding

The results of interviews with teachers show that student subjects categorized as having high and low abilities are as follows:

Table 3. Identity of research subjects

No	Student code	Student's initial name	Academic ability
1	T1	RRS	High
2	T2	SRD	High
3	R1	YC	Low
4	R2	DN	Low

a.) Data description of high ability subject (T1)

The following are the results of tests and interviews on high ability subjects (T1):

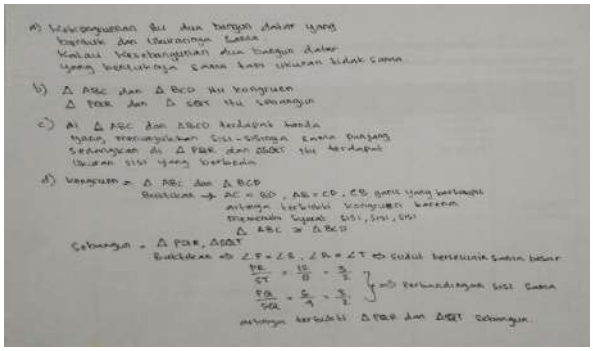


Figure 5. Answer sheet of subject T1

In this section, the data analysis of Subject T1 related to students' understanding of the concepts of congruence and similarity will be presented. Information was obtained from the test results given after the HLT implementation and in-depth interviews conducted to further explore students' understanding based on the indicators of concept understanding.

1. Restating a concept

Subject T1 has fulfilled this indicator, which can be seen from his ability to restate the concept of congruence and similarity using his own words. This is evident through his answer in question 1 (a).

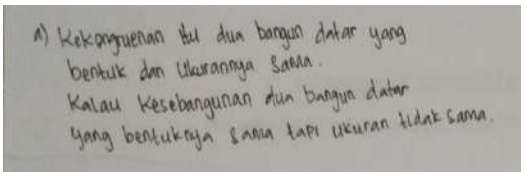


Figure 6. Defining congruence and similarity

The following are the results of the interview with subject T1:

P : "What do you think is meant by congruent flat shapes?"

T1 : "Congruent flat shapes are shapes that are exactly the same, meaning they have the same shape and the same size. So if you paste it, it can fit, covering all the parts."

P : "Does it have to be exactly the same shape? Or just the same size?"

T1 : "It has to be both. If the size is the same but the shape is different, it's not congruent. The shape and size must be the same."

Based on the test and interview results, subject T1 was able to restate the concept of congruence using his own language. He explained that two flat shapes are said to be congruent if they have the same shape and size, and can cover each other appropriately. This shows that the student not only memorized the definition, but also understood the meaning of the concept of congruence deeply.

2. Categorize objects based on certain characteristics shared by the concept and Provide examples and non-examples of the concept.

Subject T1 showed good ability in categorizing flat shapes based on the characteristics of congruence and similarity. In problems that display several pairs of flat shapes, students can categorize which ones are congruent and similarity.

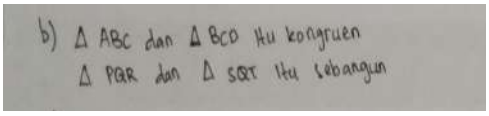


Figure 7. Choosing which ones are congruent and similarity

The following are the results of the interview with subject T1:
P : "Earlier you said triangle ABC and triangle BCD are congruent, why?"
T1 : "Because the sides are the same length and the shape is also the same. the picture has a sign that the sides are the same length."
P : "What about triangle PQR and triangle SQT?"
T1 : "They are congruent, because they have the same shape, but different sizes. So it's just similar in shape."
P: "Are all shapes that are similar in shape similarity?"
T1 : "Not necessarily. If the angles are not the same or the side comparisons are different, they are not congruent."

Based on the test and interview results, subject T1 was able to show a good understanding of the concepts of congruence and similarity through the ability to categorize objects and provide examples and non-examples of the two concepts. The student's ability to group shapes based on similarity in size, shape, and proportional side comparison, reflects mastery of the characteristics underlying the concepts of congruence and similarity. In addition, students can also correctly name and explain examples of shapes that meet the concept, and reject non-examples that do not meet these characteristics, with logical and mathematical reasons.

3. Displaying concepts in various forms of mathematical representations

Subject T1 showed the ability to display the concepts of congruence and similarity through images and mathematical symbols or signs. In the picture of the triangle provided in the posttest question, students can identify the equal side sign as a pointer that the two figures are congruent. Students also understand that different but comparable sizes show the relationship between the shapes.

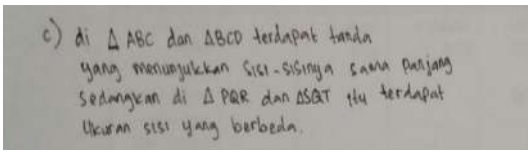


Figure 8. Providing reasoning about the concept

The following are the results of the interview with subject T1:
P : "Earlier you said the triangles were congruent, why?"
T1 : "Because in the picture there are marks on the sides. If there is the same mark on the two sides, it means they are the same length."
P : "So you see from the marks, huh?"
T1 : "Yes, so I'm sure it's congruent."
P : "For the congruent one, you said the size is different. Can you explain?"
T1 : "Yes, in the picture there are sizes, for example one side is 12 cm, the other one is 8 cm, but the shape is the same. So I said it's similarity."

The test and interview results showed that subject T1 was able to understand and display the concepts of congruence and equality through visual (picture), symbolic (equal side length sign), and numerical (side length measure) representations. The subject is able to

connect symbols or signs in the picture with the appropriate mathematical meaning, for example, the line mark indicates the same side length, as well as different but proportional sizes as a characteristic of similarity.

4. Using, utilizing, selecting certain procedures or operations and Applying problem-solving concepts or theorems

T1 students showed the ability to use appropriate mathematical procedures, as well as apply the concepts of similarity and congruence in the context of problem solving. In problems related to similarity, students calculate the comparison of corresponding sides using fractions. Meanwhile, in the congruence problem, students identify that two flat shapes are congruent by using the Side-Side-Side (SSS) rule. Students compare the lengths of three pairs of sides of the two figures and conclude that if all the corresponding sides have the same length, then the figures are congruent.

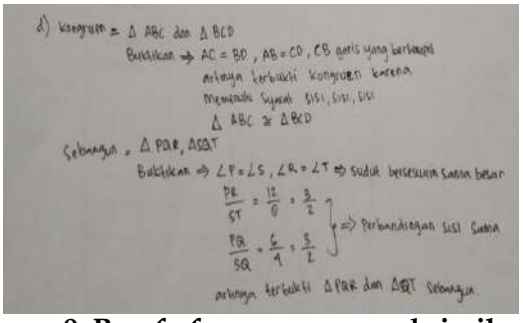


Figure 9. Proof of congruence and similarity

The following are the results of the interview with subject T1:

P : "When you find the unknown side in the congruent triangles, what method do you use?"
T1 : "I looked at the corresponding sides, then I compared them and made a fraction."
P : "What do you conclude about congruent shapes?"
T1 : "I match the lengths of the three corresponding sides. If they are the same, then they are congruent, using the S-S-S method."

From the test and interview results, it can be seen that student T1 is able to select and use appropriate mathematical procedures in solving problems related to congruence and similarity. His thought process reflects good concept understanding and procedural skills. In congruence, students apply the principle of proportional side comparison, while in symmetry, students apply the Side-Side- Side (SSS) rule consistently and logically.

b) Data description of high ability subject (T2)

The following are the test and interview results on the high ability subject (T2):

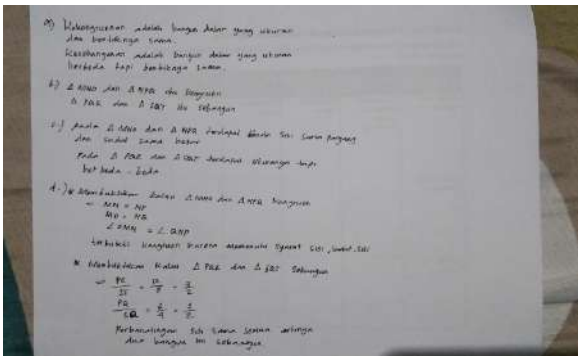


Figure 10. T2 subject's answer sheet

1. Restating a concept

Subject T2 showed an understanding of the concepts of congruence and similarity by restating the definitions of the two concepts using his own language. In the test answer, the subject wrote that the shapes are said to be congruent if they are the same shape and size, while the shapes that are congruent are the same shape but the sizes can be different.

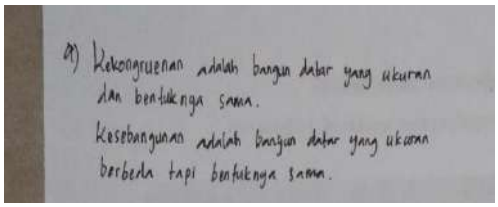


Figure 11. Defining congruence and similarity

The following are the results of the interview with subject T2:

P : "What do you think are congruent shapes?"

T2 : "Congruent is that the shapes are exactly the same, the sizes are also not different. So it's like twins."

P : "What about similarity?"

T2 : "If it's similarity, the shape is similar, but the size can be bigger or smaller."

Subject T2's statement shows that he is able to re-express mathematical concepts using his own language but still accurate in meaning. The way the student mentioned "like twins" and "similar but different sizes" showed that the student did not just memorize the definition, but understood the concept intuitively and could communicate it naturally.

2. Categorize objects based on certain characteristics shared by the concept and Provide examples and non-examples of the concept.

In answering the questions on the test, subject T2 is able to distinguish flat shapes that meet the requirements of congruence and similarity, and classify these shapes based on their characteristics. For example, students state that two shapes are congruent because they have the same side and angle sizes, while two shapes that are the same shape but different sizes are similarity.

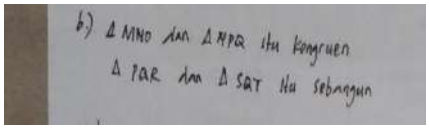


Figure 12. Choosing which ones are congruent and similarity

The following are the results of the interview with subject T2:

P : "How do you know the shapes are congruent?"

T2 : "If the sides are all the same length and the angles are the same, they are congruent."

P : "What about similarity?"

T2 : "If the shape is the same but the size is different. So it's like a smaller or bigger version of the same shape."

P : "Can you give an example?"

T2 : "For example, if two triangles are equal in size and shape, they are congruent. But if the triangles are bigger but the shape is the same, they are similarity."

Subject T2 showed the ability to identify the characteristics of flat shapes based on the concepts of congruence and congruence, and was able to group the shapes appropriately. The ability to mention examples and non-examples in a relevant and contextual manner also shows that students understand concepts not only by rote, but also functionally.

3. Displaying concepts in various forms of mathematical representations

Subject T2 showed his ability to display the concepts of congruence and similarity through visual representations. In working on the problem, he looked at the signs on the picture - such as the signs on the sides that show the same length, and the same angle - to infer congruence. The subject also mentioned that if the size of the shapes is different but the shape remains the same, then it shows congruence, which is shown through the difference in size but still maintains the shape.

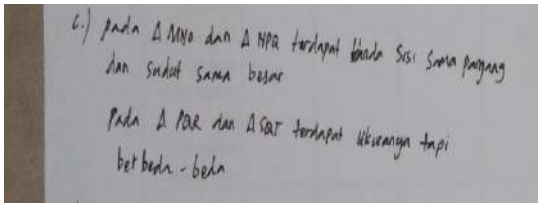


Figure 13. Giving reasons about the concept

The following are the results of the interview with subject T2:

P : "How do you know if two shapes are congruent?"

T2 : "The problem is that in the picture there is a sign that the sides are the same length and the angles are also the same size."

P : "Are they similarity?"

T2 : "If it's similarity, the size is different. But the shape is the same."

Subject T2's statement showed that he could relate the visual representation in the picture (side and angle length marks) to the concept being studied. He used the visual aspect as evidence, and this is part of the ability to present concepts in the form of non-verbal representations.

4. Using, utilizing, selecting certain procedures or operations and Applying problem-solving concepts or theorems

Subject T2 showed a good understanding in using appropriate mathematical procedures, especially in solving problems regarding congruence and similarity. In determining congruence, students calculate the comparison of corresponding sides. To determine congruence, students state that two triangles are congruent if two sides and one angle between the sides are equal, which is based on the side-angle-side condition.

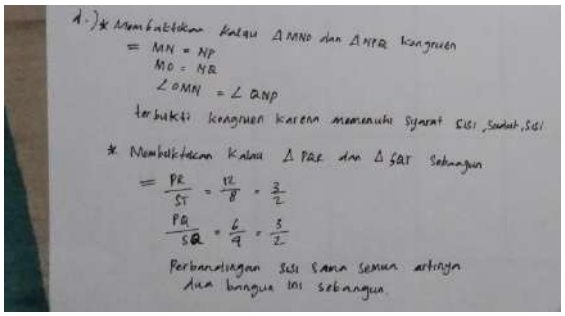


Figure 14. Proof of congruence and similarity

The following are the results of the interview with subject T2:

P : "Earlier you said the shapes are congruent, why?"

T2 : "Because the two sides are the same length and the angle between the sides is also the same. So it's side, angle, side."

P : "If the problem is similarity, how do you know?"

T2 : "I compare the side ratio. If the values are the same, it means they are similarity. You can also find the other side using cross multiplication."

Subject T2 showed good ability in using appropriate mathematical calculation procedures and applying the concepts of congruence and similarity in problem solving. In the context of congruence, students can identify pairs of corresponding sides, calculate the comparison of side lengths, and simplify the results. Meanwhile, in the congruence problem, students say that two shapes are congruent if they have two sides and one angle between the sides that are equal.

Based on the results of the data description, it is concluded that High 1 and High 2 subjects showed a thorough understanding of the concepts of congruence and similarity by fulfilling all indicators of understanding. Both were able to (1) restate the concepts of congruence and similarity in their own language precisely and clearly. They are also able to (2)classify objects or flat buildings based on characteristics that match the concept. In addition, the subjects were able to (3)give examples and non-examples of congruent and congruent shapes correctly, showing the ability to distinguish concepts critically. Both subjects can also (4)display concepts in various mathematical representations, such as pictures, symbols, and narratives based on the folklore context used in the learning. They showed the ability to (5)select and use certain procedures or operations, for example when calculating side lengths or determining the bisection of two figures, with systematic and precise steps. Finally, they were also able to (6)apply relevant concepts and theorems in solving various contextual problems given, and were able to explain their thinking process logically. This shows that high-ability subjects have strong conceptual understanding and are able to integrate mathematics with cultural contexts in a meaningful way.

c) Data description of low ability subject (R1)

The following are the results of tests and interviews on low ability subjects (R1):

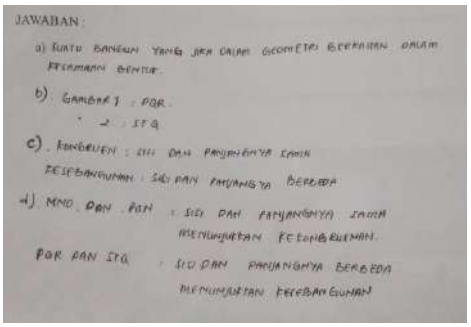


Figure 15. Answer sheet of subject R1

1. Restating a concept

Subject R1 understands that congruence and similarity are related to the shape of the shapes, but has not explicitly mentioned the mathematical characteristics that define congruence and similarity, such as size, side length, and angle magnitude.

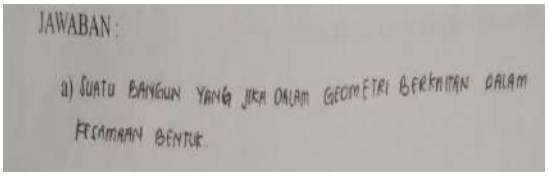


Figure 16. Defining congruence and similarity

The following are the results of the interview with subject R1:
P : "What do you think are congruent or similarity shapes?"
R1 : "Hmm... congruent is... a shape that is the same. If they are similarity... they are similar too."
P : "What distinguishes congruent and similarity shapes?"
R1 : "Congruent is the same, if similarity is like smaller or bigger, but the shape is similar."

Based on the test and interview results, it can be seen that subject R1 has an initial understanding of the concepts of congruence and similarity. He can mention that both are related to the similarity of shapes. However, R1 has not shown a complete understanding of the mathematical characteristics that distinguish the two. For example, R1 did not mention that congruent shapes must have exactly the same sides and angles, while congruent shapes only require matching shapes and angles, but with proportionally different sizes. Statements such as "same" or "similar" without linking to mathematical properties show that R1's understanding is still intuitive and visual, not conceptual and formal. This shows that in the first indicator, R1 is still at the initial understanding stage and needs assistance in linking everyday language with appropriate mathematical terms.

2. Categorizing objects based on certain characteristics shared by the concept and Providing examples and non-examples of the concept.

On the test sheet given, subject R1 was asked to categorize two flat figures based on the concepts of congruence and similarity, and determine which one is congruent and which one is congruent. However, the subject only wrote the name of the picture without providing the requested category.
In fact, conceptually, the two figures are congruent. Answers like this show that R1 has not been able to categorize objects based on the characteristics of congruence and congruence as expected.

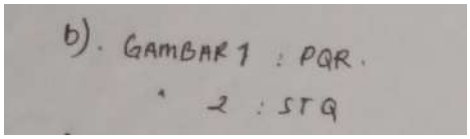


Figure 17. Choosing which ones are congruent and similarity

The following are the results of the interview with subject R1:
P : "If you look at these two triangular figures, which one do you think is congruent and which one is similarity?"
R1 : "This one is the same shape and size, it's congruent. The other one is similar but bigger, it's similarity."
P : "If there are drawings with different shapes, for example a triangle and a square, are they congruent or similarity?"
R1 : "If it's a different shape, it's not, like a square and a triangle are very different."

Although subject R1 could state that congruent figures have the same shape and size, and that congruent figures have the same shape but different sizes, the answer was still simple and based on visual observation only. In addition, when asked to explicitly categorize the images given on the test, the subject has not been able to associate the concept with the image appropriately, only writing the name of the image without determining the category of congruent or similarity. This shows that the subject's understanding of the mathematical characteristics that distinguish congruent and congruent shapes is still immature. The subject relied more on visual impressions and everyday terms ("same," "similar") without understanding formal concepts such as the similarity of sides and right angles, or the proportional comparison of sides in congruent shapes.

3. Displaying concepts in various forms of mathematical representations

Subject R1 has understood the basic difference between congruent and congruent shapes in terms of sides and length, although the explanation is still very simple and incomplete. R1 has not used other representations such as pictures, symbols, or concrete models to clarify the concept.

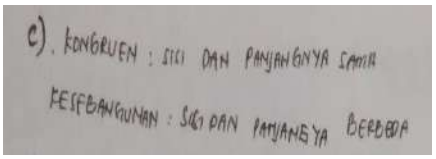


Figure 18. Giving reasons about the concept

The following are the results of the interview with subject R1:

P : "Earlier you said that these shapes are congruent. How do you know?"
R1 : "Because it looks like the same size."
P : "Did you pay attention to the signs in the picture?"
R1 : "Hmm... not really. It just looks the same shape and size."
P : "For this one you said it's similarity, why?"
R1 : "Because one is big and one is small, but the shape is similar."

Subject R1 seemed unable to utilize the available visual information mathematically, such as equal length side marks, angle marks, or side sizes, even though they were provided in the picture. He still relied on visual judgment by sight, without actually reading and interpreting the mathematical representation in the picture. Thus, although the subject could mention in general that "congruent is the same, similarity is different in size", his understanding did not yet show the ability to display or read concepts through mathematical representations as a whole, especially visual symbols in the image. This shows that R1 is still at the visual intuitive stage, and needs guidance to realize the mathematical meaning of symbols and signs in visual representations, and connect them to the concepts being studied.

4. Using, utilizing, selecting certain procedures or operations and Applying problem-solving concepts or theorems

Subject R1 did not show mathematical proof as requested in the problem. He did not mention the side or angle marks on the congruent figures, and did not compare fractions to show that the figures were similarity.

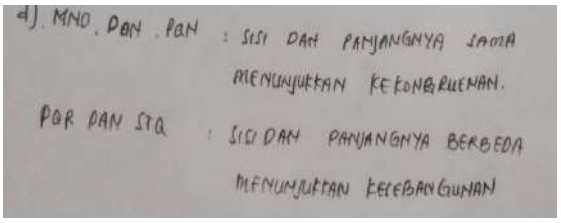


Figure 19. Proof of congruence and similarity

The following are the results of the interview with subject R1:

P : "Earlier you said these shapes are congruent, why?"
R1 : "Because the length of the sides are the same."
P : "Do you see the side marks or angle marks in the picture?"
R1 : "No, just look at the shape and size."
P : "When you say that these shapes are similarity, do you compare the sides?"
R1 : "No, I just look at one big, one small. The shapes are similar."

Subject R1 did not apply the mathematical procedures requested in the problem. He did not utilize the signs on the shapes (such as side or angle signs) as a basis for proving congruence, and did not compare fractions between sides to prove that the shapes are similarity. The subject only gave descriptive answers based on visual observations without a mathematical proof process. This shows that they do not understand how to connect the concepts of congruence and similarity with appropriate problem-solving strategies, and cannot select and use procedures (such as reading signs or comparing fractions) independently.

d) Data description of low ability subject (R2)

The following are the results of tests and interviews on low ability subjects (R2):

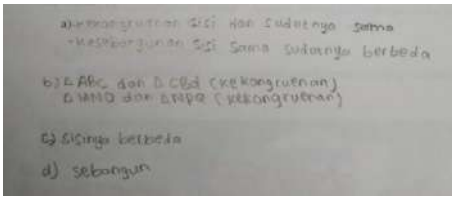


Figure 20. subject R2's answer sheet

1. Restate a concept

Understanding the concept of congruence is quite precise, students mention the same side and angle in accordance with the formal definition. However, the explanation of congruence is inaccurate. Students say: sides are equal, angles are different, when it should be: angles are equal, sides are comparable. Students' understanding is still ambiguous in distinguishing between congruence and equality, especially in the relationship between angles and sides.

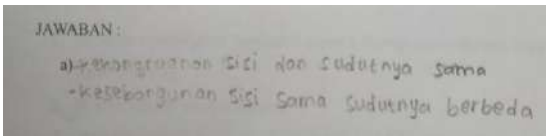


Figure 21. Defining congruence and similarity

The following are the results of the interview with subject R2:

P : What do you know about congruent flat shapes?

R2 : "Buildings with the same side and the same angle."

P : What do you think is similarity?

R2 : "The sides are the same but the angles are different."

R2's answer reflects the basic elements of congruence. Students can restate it in their own words, but still do not mention the similarity of shape and size as a whole, indicating a misconception. Students do not understand that what is the same in similarity is the angle magnitude, while the sides must be comparable (not the same length).

2. Categorize objects based on certain characteristics shared by the concept and Provide examples and non-examples of the concept.

When asked to identify congruent triangles, the subject mentioned the pair of triangles ABC and CBD, which is indeed a congruent pair. This shows that students are able to recognize shapes that have the same size and angle, although it is likely that the selection is based on visual perception alone, not due to an understanding of mathematical characteristics (such as similarity in side length and angle magnitude). However, when asked to name the congruent triangles, the subject mentioned the triangle pair MNO and NPQ, even though

they are congruent triangles, not congruent. This shows that the subject has not been able to correctly distinguish the characteristics of congruent and similarity shapes, and has not been able to conceptually categorize objects according to their characteristics.
The following are the results of the interview with subject R2:

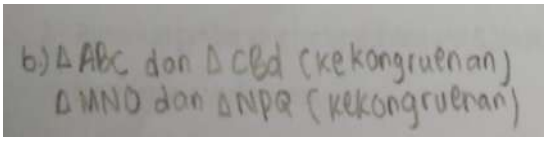


Figure 22. Choosing which ones are congruent and similarity

P : "In the picture there are several triangles. According to you, which triangles are congruent?"
R2 : "The ABC and CBD ... that seems congruent."
P : "Which ones are similarity?"
R2 : "MNO and NPQ, because they are also similar and the same size"
P : "Do you know the difference between congruence and similarity?"
R2 : "If they are the same shape and size, they are congruent. If it's similarity, I think it's similar like that."

From the test and interview results, it can be interpreted that the subject was able to correctly identify pairs of congruent triangles based on visual similarity and perception of the same size. When asked to name pairs of similarity triangles, the subject chose pairs of triangles that were actually congruent. This shows that the subject does not yet understand the fundamental difference between the concepts of congruent and similarity mathematically.

3. Displaying concepts in various forms of mathematical representation

Subject R2 only said "the sides are different," showing that he has not been able to utilize images to explain mathematical relationships such as side comparison and angle similarity in detail. The subject relied more on simple visual observations without using other mathematical representations, such as calculations of side ratios or explanations of angles.
The following are the results of the interview with subject R2:

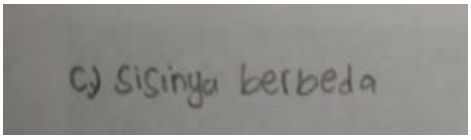


Figure 23. Giving reasons about the concept

P : "Try to explain, why triangles ABC and CBD are congruent?"
R2 :"The sides are the same."
P : "What about triangles MNO and NPQ? Are they similarity?"
R2 :"Yes, but the sides are different."
P : "Can you explain in more detail why they are congruent even though the sides are different?"
R2 : "Because the sides are different."

Subject R2's answer which only mentions "the sides are different" as the reason why the triangles are congruent shows limitations in expressing the concept of similarity mathematically. The subject has not been able to relate the concept of congruent with more complete characteristics, such as the comparison of comparable side lengths and the similarity of angle magnitudes. Similarly, in explaining congruence, they only highlighted the similarity of sides without being able to explain mathematically the relationship underlying congruence, such as the similarity of side lengths in pairs and the similarity of angle magnitudes. This indicates that students have not been able to display the concepts of congruent and similarity in various forms of mathematical representation.

4. Use, utilize, select certain procedures or operations and Apply problem-solving concepts or theorems

Students only answer "congruent" on the question of proving congruence and similarity, therefore the interview stage is carried out to deepen the understanding of subject R2.

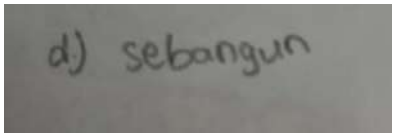


Figure 24. Proof of congruence and similarity

The following are the results of the interview with subject R2:
P : "How do you prove that two triangles are congruent?"
R2 : "Hmm... if they are the same size, they are congruent."
P : "What does it mean if they are the same size? Can you explain in more detail?"
R2 : "Yes... the sides are all the same."
P : "Now, how do you prove that two triangles are similarity?"
R2 : "Look at the picture... the sides are different, but the shapes are similar."
P : "Have you ever learned about comparing sides or angles to prove congruence?"
R2 : "Yes, I have. But I kind of forgot how to compare."

From the test and interview results, subject R2 showed an initial understanding that congruence is related to the similarity of shape and size, and congruence is related to the same shape but different sizes. However, the subject has not been able to explain or apply the proof procedure mathematically.

Based on the results of the data description, it is concluded that low ability students, namely Low 1 and Low 2 subjects, were only able to fulfill two of the six indicators of understanding the concepts of congruence and similarity. Both of them were able to (1) restate the concepts of congruence and similarity with simple sentences and could (2) classify flat buildings based on characteristics that match the concept. However, their abilities were still limited and did not show a thorough understanding. Low subjects were not able to provide examples and non- examples correctly, and had difficulty in using various forms of mathematical representations, such as appropriate symbols or pictures. They were also unable to select appropriate mathematical procedures or operations in solving problems, and could not apply concepts or theorems effectively in problem solving. In addition, they tend to be unable to provide logical reasons for the answers given. This shows that their understanding is still at an early stage and requires further assistance in order to develop deeper and more structured mathematical thinking skills.

Based on the results of the analysis, it is known that students' understanding of the concepts of congruence and similarity shows variations that are influenced by students' initial abilities and the effectiveness of the folklore-based HLT implemented. Students with high ability tended to be able to identify the properties of congruent and congruent shapes correctly, and could relate the context of the folklore "Asal Usul Watu Ulo" to the representation of similar flat shapes. In contrast, low ability students showed difficulty in distinguishing between the concepts of congruence and similarity. They tend to equate two congruent shapes to be congruent, without considering the difference in size. This finding is in line with research by Safitri (2020) which states that "often students cannot distinguish that two similarity flat shapes are not necessarily congruent, and that two congruent flat shapes are definitely similarity." This shows that misconceptions about these two concepts are still strong, especially among low ability students.

On the other hand, the implementation of folklore-based HLT in this study contributed positively to students' understanding, because learning took place in a familiar

context. This result is in line with research by Sary (2017) who designed HLT using folklore for value comparison materials. She found that the context of local culture can help students understand math concepts more meaningfully. This research is also reinforced by the opinion of Nova & Putra (2022) which states that "cultural and social values that develop in the community will help the process of understanding mathematics at school". This indicates that the incorporation of cultural elements in mathematics learning not only strengthens the relevance of the material, but also builds students' emotional and cognitive engagement with the concepts learned.

CONCLUSION

The folklore-based HLT design proved to be effective in helping students understand the concepts of congruence and congruence. Folktales provide a meaningful context that is close to students' lives, so the learning process becomes more interesting and meaningful. Students with high ability tended to be able to follow the flow of learning activities in the HLT well. They were able to discover the properties of similarity and congruence by themselves through explorative activities, and could provide logical explanations for the relationships between flat shapes. Students with low ability showed an increase in understanding, although there were still difficulties in distinguishing between congruent and similarity shapes. They tend to assume that all similarity shapes must be congruent. Therefore, it is necessary to provide more intensive assistance, as well as using concrete and visual media more explicitly. In addition, teachers should provide reinforcement of concepts repeatedly and gradually, so that students can more easily distinguish between congruence and similarity. For future researchers, it is recommended to develop HLT with variations of other folklore contexts or on different mathematics materials, so that there are more references to contextual learning designs that support students' concept understanding at various ability levels.

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