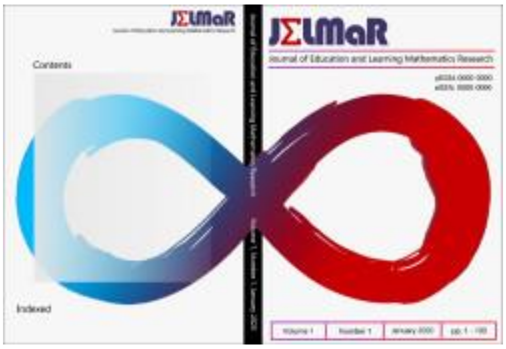

Journal of Education and Learning Mathematics Research (JELMaR)

Online ISSN : 2715-9787
Print ISSN : 2715-8535
Journal Homepage : http://jelmar.wisnuwardhana.ac.id/index.php/jelmar/index

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To cite this article: Kar, R., & Shaw, A. (2020). Analysis The Barrier of E-learning in Mathematics Using Type-2 Fuzzy Data. *Journal of Education and Learning Mathematics Research (JELMaR)*, *1*(2), 58-73. https://doi.org/10.37303/jelmar.v1i2.31

To link this article : https://doi.org/10.37303/jelmar.v1i2.31

Publisher
Department of Mathematics Education,
Faculty of Teacher Training and Education,
Universitas Wisnuwardhana Malang

Journal of Education and Learning Mathematics Research (JELMaR)
Volume: 1, Number: 2, November 2020, pp. 58-73
p-ISSN: 2715-8535, e-ISSN: 2715-9787

Received: Juni 2020 *Revised: September 2020* *Accepted: November 2020* *Published: November 2020*

Analysis the Barrier of E-Learning in Mathematics Using Type-2 Fuzzy Data

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***Abstract**: E-Learning is mean to learn all round educational subjects taking assist of modern technology of the online. E-learning is also a big platform to learn mathematics. But in the current public health crisis, we are all working quickly to move our classes out of the classroom. Fortunately, even if online teaching and learning are new to all of us, some uncertainties are there exists. According to modern view uncertainty is considered essential to science and technology, it is not only the unavoidable plague but also it has impact a great utility. Generally, fuzzy sets are used to analyse fuzzy system reliability. To analyse the fuzzy system reliability, the reliability of each component of the system is considered as a Triangular intuitionistic Type 2 fuzzy number (TIT2FN).*

Keyword: Triangular intuitionistic Type 2 fuzzy number (TIT2FN), System reliability, Parallel system, Series system, e-learning, e-learning mathematics

INTRODUCTION

Necessity of E-Learning:

In present busy world many a student cannot attend the classroom but in the online system they need not to do so. They can acquire their subjects matter through technology completely online. There are lot of advantages. A student can complete his course, take his degree in different subjects from online or internet where lot of trained and experienced teachers, professors deliver lecture as per the demand of learners. Here is no need of DVD, CD or a T.V. but although students have chance to talk to your teachers, professors and students. Sometimes the lectures are recorded to hear in future. The teacher can easily justify students ability, brightness, lack of knowledge etc and their grade will be given accordingly to their ability. It is proved that online training is a well method for modern time to get a successful life.

Review on Reliability:

It is known that conventional reliability analyses using probabilities have been found be inadequate in handling uncertainty of failure data and modelling. Onisawa and Kacprzyk (1995) used the concept of fuzzy approach f evaluation of the reliability of a system to overcome the problem. The discipline of the reliability engineering encompasses a number of different activities is pointed out by Kaufmann and Gupta (1988). Cai et al. (1991) represented the following two fundamental assumptions in the conventional reliability theory like **Binary state assumptions: Probability assumptions**. The system failure engineering and its use of the fuzzy logic was described by Cai et al. (1993). Cheng and Mon (1993) presented a method for analysing fuzzy system reliability using fuzzy number arithmetic operations. Using the concept of probist reliability as a triangular fuzzy number in dynamic reliability evaluation of deteriorating systems presented by Verma et al. (2004).

Review on Type-2 Fuzzy Number:

The theory and design of interval type-2 fuzzy logic systems (FLSs) had been introduced by Liang and Mendel in the year of 2000. The concept of type-2 intuitionistic fuzzy sets under type-2 fuzzy sets and intuitionistic fuzzy sets was introduced by **Tao and Jian [2012]**. Hu et. al [2013] had proposed a new approach to solve multi-criteria decision making (MCDM) problems based on the form of interval type-2 fuzzy number. Mazandarani and Najariyan[2014] had defined a differentiability of the type-2 fuzzy number-valued functions. A new approach had been applied by Wang et. al [2015] for solving multi-criteria group decision-making (MCGDM) problems, which is described by trapezoidal interval type-2 fuzzy numbers (IT2FNs). An easy to approach to the problems of transportation had described by Chhibber et. al[2019] where incentre of centroids has been employed to convert trapezoidal fuzzy transportation problem of type 1 and type-2 both into crisp one. Senthil Kumar [2020] had designed a transportation problem where he used intuitionistic fuzzy number for supplies, demands.

Motivation:

Many research papers are available today in this world but discussion on barrier of e-learning especially on mathematics is not available using fuzzy data. Today e-learning is well developed discipline and has branched out into specialised areas such as mathematics, science, social-science, language group etc. In this paper we mainly utilise the uncertainty properties of fuzzy data to represent the failure of e-learning in mathematics by reliability system.

Novelties:

Some new interest and new work have done by our self which is mentioned below:

i. Represent all most of the major barrier in e-learning of mathematics by reliability system using fuzzy data.
ii. Try to utilise the properties of Triangular intuitionistic Type 2 fuzzy number to solve the above mentioned reliability problem.
iii. Described imprecise reliability both of series and parallel systems using Triangular intuitionistic Type 2 fuzzy number.

Frame a problem of reliability of major barrier in e-learning of mathematics with imprecise reliability components by Triangular intuitionistic Type 2 fuzzy number.

Preliminaries E-Learning

Learning includes the physical, social and pedagogical context in which learning will be taken place. A well environment increases the interest of our learners. It offers all students, teachers the strength of their minds with connectedness. It also offers to talk directly or indirectly to all members of these fields to practise more and study more.

For Students: Every student can contact one by one with their teachers. They can get assist of reference books, school basis teaching and advance education in mathematics.

For Parents: their sons and daughters can get a best theory for learning with minimum expenditure without sending them out of doors. Their H.W and C.W can be done and in the nick of time examinations are taken steps by steps.

For Teachers: A teacher loves invaluable collections of papers in mathematics and he gives a net picture to his students who desire to know more and more about mathematics and take a challenge in math-world. A teacher is well trained about e-learning system.

E-learning in Mathematics

In mathematics e-learning education provide interactive videos, can make mathematics easier and more enjoyable for solving any complex mathematics. Problems are displayed through visual and audio formats on a stimulating web-based interface. All these are done by our qualified and experienced teachers. Learning mathematics online has a range of advantages. Students can spend more time on specific subjects than they could in the traditional classroom, allowing them to learn mathematics at the perfect pace for information retention.

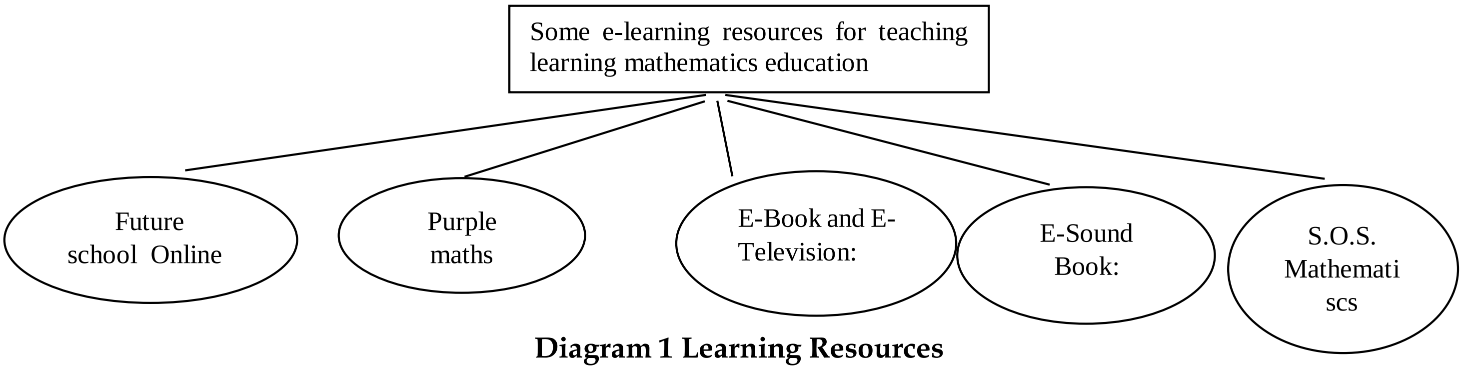

Diagram 1 Learning Resources

Challenges in E-Learning:

In the present condition of the word this system is very needful all over the earth and this one of ways to learn perfectly. But some bars are also present now days. Challenges must be taken by the teachers, students and everyone who are linked with on-line education, extraordinarily who wants to get a perfect knowledge in math. Many points are applicable but most of them can be rejected by taking the challenge both teachers and students. We have already showed the issues and can be divided four types and they are followed:

Students: The objective is to have online learning as one of the important ways for students to become successful in learning how to do mathematics as well as understand, appreciate and apply mathematics. This creates many challenges. Here the issues are as follows:

Table 1 Student Issues 1

Serial no.	Issues
1.	It is dependent on student's skill, understanding and thoroughly knowledge of online technology. Arithmetic, algebra, graphs and spreadsheets software is used extensively in this system according to age groups. Teaching is given in a group but thoroughly knowledge of medium is needed.
2.	Students display different learning styles when they learn mathematics. Understanding his merit real judgment is required for a teacher, how to guide him a difficult sums.
3.	Students want different versions of on-line system. They may require working on their math course.
4.	They need to ready, equal access to the internet network in this system.
5.	There are bars in this environment but they must contact with their teachers and other same class students.

6.	All students desire a healthy environment to take their courses.

Teacher: The teacher's role changes to more of a facilitator and the degree of change varies depending on the type of online learning. Teachers need to maintain their professionalism of quality fundamental pedagogical practices in teaching mathematics online. Here the issues are as follows:

Table 2 Issues 2

Serial no.	Issues
1.	It is paramount that an online mathematics course be thoroughly organized and developed ahead of time.
2.	Assessment strategies are altered to fit online learning in mathematics.
3.	It is essential to have research into matters that arise for discussion such as the ideal class size, the teacher's role and the use of textbooks.
4.	: Instructional design in a technological environment takes on a new role in remote teaching.
5.	A sharing of information with other teachers becomes even more important in this new environment.
6.	There could be an issue with society's perception that the role of students and their teachers has completely changed.

Learning Environment: Partnership of experts in discipline, thoroughly skill etc is essential in this system. There is a great need from student side to support the authority for their own requirement. Mathematics students are also need to know various ways how to solve different questions in an ideal environment with joy and happiness. Here the issues are as follows:

Table 3 Issues 3

Serial no.	Issues
1.	There is a monopoly of certain online technological tools that may harm the system.
2.	Decisions are made as to the degree on which a course provides.
3.	Given the time for the ideal environment for teachers as well as for students.
4.	In the nick of the time needful materials are needed for the demand of time.
5.	Everyone always be helpful to each others.

Mathematics: The educational system needs to stay true to the subject matter of mathematics. The role of pedagogy must be in control not technology or the pursuit of efficiency. The subject matter of mathematics should continue to stay all-important and not be diminished by the necessity of technology in on-line learning. Here the issues are as follows:

Table 4 Issues 4

Serial no.	Issues
1.	Rich learning tasks are important in developing mathematical concepts.
2.	Students should be able to easily communicate mathematics electronically.
3.	Best practices in a classroom may not adapt easily to online learning of mathematics.
4.	Appropriate software is needed to translate captured graphics.
5.	Inadequate different designing mathematics e-learning courses for different

Triangular Intuitionistic Type 2 Fuzzy Number

Definition: A TIT2FN $\tilde{A}^{\psi}_{IFN}$ is an IFN in R with the following membership function $\mu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)$ and non membership function $\nu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)$

$$\mu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)=\begin{cases}\varpi^{\psi}\dfrac{\hat{x}-a^{\psi}{}_1}{b^{\psi}{}_1-a^{\psi}{}_1}, a^{\psi}{}_1\le\hat{x}\le b^{\psi}{}_1\\ \varpi^{\psi} \quad ,\hat{x}=b^{\psi}{}_1\\ \varpi^{\psi}\dfrac{c^{\psi}{}_1-\hat{x}}{c^{\psi}{}_1-b^{\psi}{}_1}, b^{\psi}{}_1\le\hat{x}\le c^{\psi}{}_1\\ 0 \quad ,otherwise\end{cases} \text{ and } \nu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)=\begin{cases}\varpi^{\psi}\dfrac{b^{\psi}{}_1-\hat{x}}{b^{\psi}{}_1-a^{\psi/}{}_1}, a^{\psi/}{}_1\le\hat{x}\le b^{\psi}{}_1\\ 0 \quad ,\hat{x}=b^{\psi}{}_1\\ \varpi^{\psi}\dfrac{\hat{x}-b^{\psi}{}_1}{c^{\psi/}{}_1-b^{\psi}{}_1}, c^{\psi/}{}_1\le\hat{x}\le c^{\psi}{}_1\\ \varpi^{\psi} \quad ,otherwise\end{cases}$$

Where $a^{\psi/}{}_1<a^{\psi}{}_1<b^{\psi}{}_1<c^{\psi}{}_1<c^{\psi/}{}_1$ and $\mu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right),\nu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)\le 0.5$ for $\mu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)=\nu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right)\forall\hat{x}\in\Re$

This TIT2FN is denoted by $\tilde{A}^{\psi}_{TIT2FN}=\left(a^{\psi}{}_1,b^{\psi}{}_1,c^{\psi}{}_1;\varpi^{\psi}\right)\left(a^{\psi/}{}_1,b^{\psi}{}_1,c^{\psi/}{}_1;\varpi^{\psi}\right)$.

Some arithmetic operations of Type-2 Intuitionistic Fuzzy Number based on cuts method:

Properties 1 Multiplication of TIT2FN by Crisp Number:

If TIT2FN $\tilde{A}^{\psi}_{TIT2FN}=\left(a^{\psi}{}_1,b^{\psi}{}_1,c^{\psi}{}_1;\varpi^{\psi}_1\right)\left(a^{\psi/}{}_1,b^{\psi}{}_1,c^{\psi/}{}_1;\varpi^{\psi}_1\right)$ and $\hat{y}=ka^{\psi}\left(k>0\right)$, then $\tilde{Y}_{TIT2FN}=k\,\tilde{A}^{\psi}_{TIT2FN}$ is a TIT2FN $\left(ka^{\psi}{}_1,kb^{\psi}{}_1,,kc^{\psi}{}_1;\varpi^{\psi}\right)\left(ka^{\psi/}{}_1,kb^{\psi}{}_1,kc^{\psi/}{}_1;\varpi^{\psi}\right)$.

If $\hat{y} = ka^{\psi}\ (k<0)$ then $\tilde{Y}_{TIT2FN} = k\,\tilde{A}^{\psi}_{TIT2FN}$ is a TT2IFN $\left(kc^{\psi}{}_1, kb^{\psi}{}_1, ka^{\psi/}{}_1; \varpi^{\psi}\right)\left(kc^{\psi}{}_1, kb^{\psi}{}_1, ka^{\psi/}{}_1; \varpi^{\psi}\right)$.

Properties 2 Addition of Two TIT2FN:

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1, b^{\psi}{}_1, c^{\psi}{}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}{}_1, b^{\psi}{}_1, c^{\psi/}{}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_2, b^{\psi}{}_2, c^{\psi}{}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}{}_2, b^{\psi}{}_2, c^{\psi/}{}_2; \varpi^{\psi}_2\right)$ are two TIT2FN then $\tilde{C}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN} \oplus \tilde{B}^{\psi}_{TIT2FN}$ is also TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \oplus \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 + a^{\psi}{}_2, b^{\psi}{}_1 + b^{\psi}{}_2, c^{\psi}{}_1 + c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}{}_1 + a^{\psi/}{}_2, b^{\psi}{}_1 + b^{\psi}{}_2, c^{\psi/}{}_1 + c^{\psi/}{}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \le 1; \varpi^{\psi} = \min\left(\varpi^{\psi}_1, \varpi^{\psi}_2\right)$

Properties 3 Subtraction of Two TIT2FN:

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1, b^{\psi}{}_1, c^{\psi}{}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}{}_1, b^{\psi}{}_1, c^{\psi/}{}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_2, b^{\psi}{}_2, c^{\psi}{}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}{}_2, b^{\psi}{}_2, c^{\psi/}{}_2; \varpi^{\psi}_2\right)$ are two TIT2FN then $\tilde{C}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN}\ \Theta\ \tilde{B}^{\psi}_{TIT2FN}$ is also TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN}\ \Theta\ \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 - a^{\psi}{}_2, b^{\psi}{}_1 - b^{\psi}{}_2, c^{\psi}{}_1 - c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}{}_1 - a^{\psi/}{}_2, b^{\psi}{}_1 - b^{\psi}{}_2, c^{\psi/}{}_1 - c^{\psi/}{}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \le 1; \varpi^{\psi} = \min\left(\varpi^{\psi}_1, \varpi^{\psi}_2\right)$

Properties 4 Multiplication of Two TIT2FN:

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1, b^{\psi}{}_1, c^{\psi}{}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}{}_1, b^{\psi}{}_1, c^{\psi/}{}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_2, b^{\psi}{}_2, c^{\psi}{}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}{}_2, b^{\psi}{}_2, c^{\psi/}{}_2; \varpi^{\psi}_2\right)$ are two TIT2FN then $\tilde{P}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN} \otimes \tilde{B}^{\psi}_{TIT2FN}$ is an approximated TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \otimes \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 a^{\psi}{}_2, b^{\psi}{}_1 b^{\psi}{}_2, c^{\psi}{}_1 c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}{}_1 a^{\psi/}{}_2, b^{\psi}{}_1 b^{\psi}{}_2, c^{\psi/}{}_1 c^{\psi/}{}_2; \varpi^{\psi}\right)$$

Where

$$0 < \varpi^{\psi} \leq 1; \varpi^{\psi} = \min\left(\varpi_1^{\psi}, \varpi_2^{\psi}\right)$$

Properties 5 Division of Two TIT2FN:

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1, b^{\psi}{}_1, c^{\psi}{}_1; \varpi_1^{\psi}\right)\left(a^{\psi/}{}_1, b^{\psi}{}_1, c^{\psi/}{}_1; \varpi_1^{\psi}\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_2, b^{\psi}{}_2, c^{\psi}{}_2; \varpi_2^{\psi}\right)\left(a^{\psi/}{}_2, b^{\psi}{}_2, c^{\psi/}{}_2; \varpi_2^{\psi}\right)$ are two TIFN then $\tilde{P}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN} \div \tilde{B}^{\psi}_{TIT2FN}$ is an approximated TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \div \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 / a^{\psi}{}_2, b^{\psi}{}_1 / b^{\psi}{}_2, c^{\psi}{}_1 / c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}_1 / a^{\psi/}_2, b^{\psi}{}_1 / b^{\psi}{}_2, c^{\psi/}_1 / c^{\psi/}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \leq 1; \varpi^{\psi} = \min\left(\varpi_1^{\psi}, \varpi_2^{\psi}\right)$

Imprecise reliability of series and parallel systems using arithmetic operations or Triangular Intuitionistic Type2 Fuzzy Numbers:

The imprecise reliability of a series and a parallel system present here. Triangular Intuitionistic Type2 Fuzzy numbers are used to represent the reliability of each component of the systems.

Series System

Let us consider a series system consisting of n components. Here we want to show the intuitionistic fuzzy reliability of $\tilde{R}_{ss}^{\tau_{ss}}$. This can be evaluated by using the expression as follows:

$$\tilde{R}^{\tau_{ss}}{}_{ss} = \tilde{R}_1^{\tau_{ss}} \otimes \tilde{R}_2^{\tau_{ss}} \otimes \ldots\ldots. \otimes \tilde{R}_n^{\tau_{ss}} = \left\{\left(\hat{r}_{11}, \hat{r}_{12}, \hat{r}_{13}; \omega_1\right)\left(\hat{r}^{/}_{11}, \hat{r}_{12}, \hat{r}^{/}_{13}; \omega_1\right) \otimes \left(\hat{r}_{21}, \hat{r}_{22}, \hat{r}_{23}; \omega_2\right)\left(\hat{r}^{/}_{21}, \hat{r}_{22}, \hat{r}^{/}_{23}; \omega_2\right)\right.$$

$$\left. \otimes \ldots\ldots\ldots\ldots\ldots\ldots\ldots \otimes \left(\hat{r}_{n1}, \hat{r}_{n2}, \hat{r}_{n3}; \omega_n\right)\left(\hat{r}^{/}_{n1}, \hat{r}_{n2}, \hat{r}^{/}_{n3}; \omega_n\right)\right\}$$

The approximated to a TIT2FN can be written as;

$$\cong \left(\prod_{j^/=1}^{n} \hat{r}_{j^/1}, \prod_{j^/=1}^{n} \hat{r}_{j^/2}, \prod_{j^/=1}^{n} \hat{r}_{j^/3}; \omega\right)\left(\prod_{j^/=1}^{n} \hat{r}^{/}_{j^/1}, \prod_{j^/=1}^{n} \hat{r}_{j^/2}, \prod_{j^/=1}^{n} \hat{r}^{/}_{j^/3}; \omega\right)$$

Where $\tilde{R}_{j^/}^{\tau_{ss}} = \left(\hat{r}_{j^/1}, \hat{r}_{j^/2}, \hat{r}_{j^/3}; \omega\right)\left(\hat{r}^{/}_{j^/1}, \hat{r}_{j^/2}, \hat{r}^{/}_{j^/3}; \omega\right)$ represent the intuitionistic fuzzy reliability of the jth component for j/=1,2,......,n.

Parallel System:

Let us consider a parallel system consisting of n components. The evaluation of fuzzy reliability $\tilde{R}_{ps}^{i\tau_{ps}}$ of the parallel system is described by the following expression as follows:

$$\tilde{R}^{i^{/}\tau_{ps}}_{ps} = 1\Theta\prod_{j^{/}=1}^{n}\left(1-\tilde{R}^{i^{/}\tau_{ps}}_{j^{/}}\right) = 1\Theta\left[\left(1\Theta\left(\hat{r}_{11},\hat{r}_{12},\hat{r}_{13};\omega_1\right)\left(\hat{r}^{/}_{11},\hat{r}_{12},\hat{r}^{/}_{13};\omega_1\right)\right)\ldots\ldots\ldots \left(1\Theta\left(\hat{r}_{n1},\hat{r}_{n2},\hat{r}_{n3};\omega_n\right)\left(\hat{r}^{/}_{n1},\hat{r}_{n2},\hat{r}^{/}_{n3};\omega_n\right)\right)\right]$$

It is an approximated to a TIT2FN, Where $\tilde{R}^{i\tau_{ps}}_{j^{/}} = \left(\hat{r}_{j^{/}1},\hat{r}_{j^{/}2},\hat{r}_{j^{/}3};\omega\right)\left(\hat{r}^{/}_{j^{/}1},\hat{r}_{j^{/}2},\hat{r}^{/}_{j^{/}3};\omega\right)$ is an intuitionistic fuzzy reliability of the jth component for $j^{/}=1,2,\ldots\ldots,n$.

Parallel-Series System :

Consider a parallel-series system consisting of m branches connected in parallel. The evaluation of fuzzy reliability $\tilde{R}_{pss}^{i\tau_{pss}}$ of the parallel-series system can be evaluated as follows:

$$\tilde{R}^{i^{/}\tau_{pss}}_{pss} = \left[1-\prod_{k^{/}=1}^{m}\left(1-\left(\prod_{i^{/}=1}^{n}x_{k^{/}i^{/}}\right)\right),1-\prod_{k^{/}=1}^{m}\left(1-\left(\prod_{i=1}^{n}y_{k^{/}i^{/}}\right)\right),1-\prod_{k^{/}=1}^{m}\left(1-\left(\prod_{i=1}^{n}z_{k^{/}i^{/}}\right)\right);\omega\right]\left[1-\prod_{k^{/}=1}^{m}\left(1-\left(\prod_{i^{/}=1}^{n}x^{/}_{k^{/}i^{/}}\right)\right),1-\prod_{k^{/}=1}^{m}\left(1-\left(\prod_{i^{/}=1}^{n}y_{k^{/}i^{/}}\right)\right),1-\prod_{k^{/}=1}^{m}\left(1-\left(\prod_{i^{/}=1}^{n}z^{/}_{k^{/}i^{/}}\right)\right);\omega\right]$$

Where $\tilde{R}^{i^{/}\tau_{sps}}_{i^{/}k^{/}} = \left(x_{i^{/}k^{/}},y_{i^{/}k^{/}},z_{i^{/}k^{/}};\omega\right)\left(x^{/}_{i^{/}k^{/}},y_{i^{/}k^{/}},z^{/}_{i^{/}k^{/}};\omega\right)$ represents the reliability of the ith component at kth branch.

Series-Parallel System:

Consider a series-parallel system consisting of n stages connected in series and each stage contains m components. The fuzzy reliability $\tilde{R}_{sps}^{i\tau_{sps}}$ of the series-parallel system can be evaluated as follows:

$$\tilde{R}^{i\tau_{sps}}_{sps} = \left[\left(\prod_{k^{/}=1}^{n}\left(1-\left(\prod_{i^{/}=1}^{m}x_{i^{/}k^{/}}\right)\right),\prod_{k^{/}=1}^{n}\left(1-\left(\prod_{i^{/}=1}^{m}y_{i^{/}k^{/}}\right)\right),\prod_{k^{/}=1}^{n}\left(1-\left(\prod_{i^{/}=1}^{m}z_{i^{/}k^{/}}\right)\right)\right);\omega\right]\left[\left(\prod_{k^{/}=1}^{n}\left(1-\left(\prod_{i^{/}=1}^{m}x^{/}_{i^{/}k^{/}}\right)\right),\prod_{k^{/}=1}^{n}\left(1-\left(\prod_{i^{/}=1}^{m}y_{i^{/}k^{/}}\right)\right),\prod_{k^{/}=1}^{n}\left(1-\left(\prod_{i^{/}=1}^{m}z^{/}_{i^{/}k^{/}}\right)\right)\right);\omega\right]$$

Where $\tilde{R}^{i^{/}\tau_{sps}}_{i^{/}k^{/}} = \left(x_{i^{/}k^{/}},y_{i^{/}k^{/}},z_{i^{/}k^{/}};\omega\right)\left(x^{/}_{i^{/}k^{/}},y_{i^{/}k^{/}},z^{/}_{i^{/}k^{/}};\omega\right)$ represent the reliability of the ith component at kth stage.

Application with Discussion:
Calculation of system barrier in e-learning of Mathematics using Triangular Intuitionistic Type2 Fuzzy Numbers:

Barrier in e-learning of Mathematics depends on different facts. The facts are described below. There are five sub factors of each of facts. The fault-tree of barrier in e-learning of Mathematics is shown in the figure 1.

$\tilde{F}_S$ =Student

$\tilde{F}_{S_1}$ =Inability of students in certain skills such as the ability to read and be technologically proficient.

$\tilde{F}_{S_2}$ =Challenge to accommodate the student needs.

$\tilde{F}_{S_3}$ =Students choice based instructional deign missing for course or modules.

$\tilde{F}_{S_4}$ =Inequitable availability of network of students.

$\tilde{F}_{S_5}$ =The mathematics curriculum must be upheld and paramount in ensuring quality mathematical learning.

$\tilde{F}_T$ =Teacher

$\tilde{F}_{T_1}$ =Low emphasis on teachers' practice such as the advance preparation and complete lesson plan.

$\tilde{F}_{T_2}$ =Unfit assessment strategies for online mathematics learning.

$\tilde{F}_{T_3}$ =Ideal class size and uses of text books is a big issue.

$\tilde{F}_{T_4}$ =Teacher skill and knowledge in instructional design is needed.

$\tilde{F}_{T_5}$ =Less connection with other teachers for gathering of best practices for experienced role model.

$\tilde{F}_{LE}$ =Learning environment

$\tilde{F}_{LE_1}$ =Monopoly of certain online technological tools that may influence the choice of pedagogy.

$\tilde{F}_{LE_2}$ =Decisions are made as to the degree to which online learning is included in a course.

$\tilde{F}_{LE_3}$ =Educators are pressured to take quick decision on the timing and the extent of online learning.

$\tilde{F}_{LE_4}$ =Students require ease in navigating through courses.

$\tilde{F}_{LE_5}$ =There is a need for educators to share and learn from each other in this environment.

$\tilde{F}_M$ =Mathematics

$\tilde{F}_{M_1}$ =Online technology tools need to support exploration and discussion in mathematics.

$\tilde{F}_{M_2}$ =Less collaborative communicative system and easily manipulated mathematical objects in the online environment.

$\tilde{F}_{M_3}$ =Best practices in the classroom may not adapt easily to online learning of mathematics.

$\tilde{F}_{M_4}$ =Appropriate software is needed to translate captured graphics.

$\tilde{F}_{M_5}$ =Inadequate different designing mathematics e-learning courses for different generations.

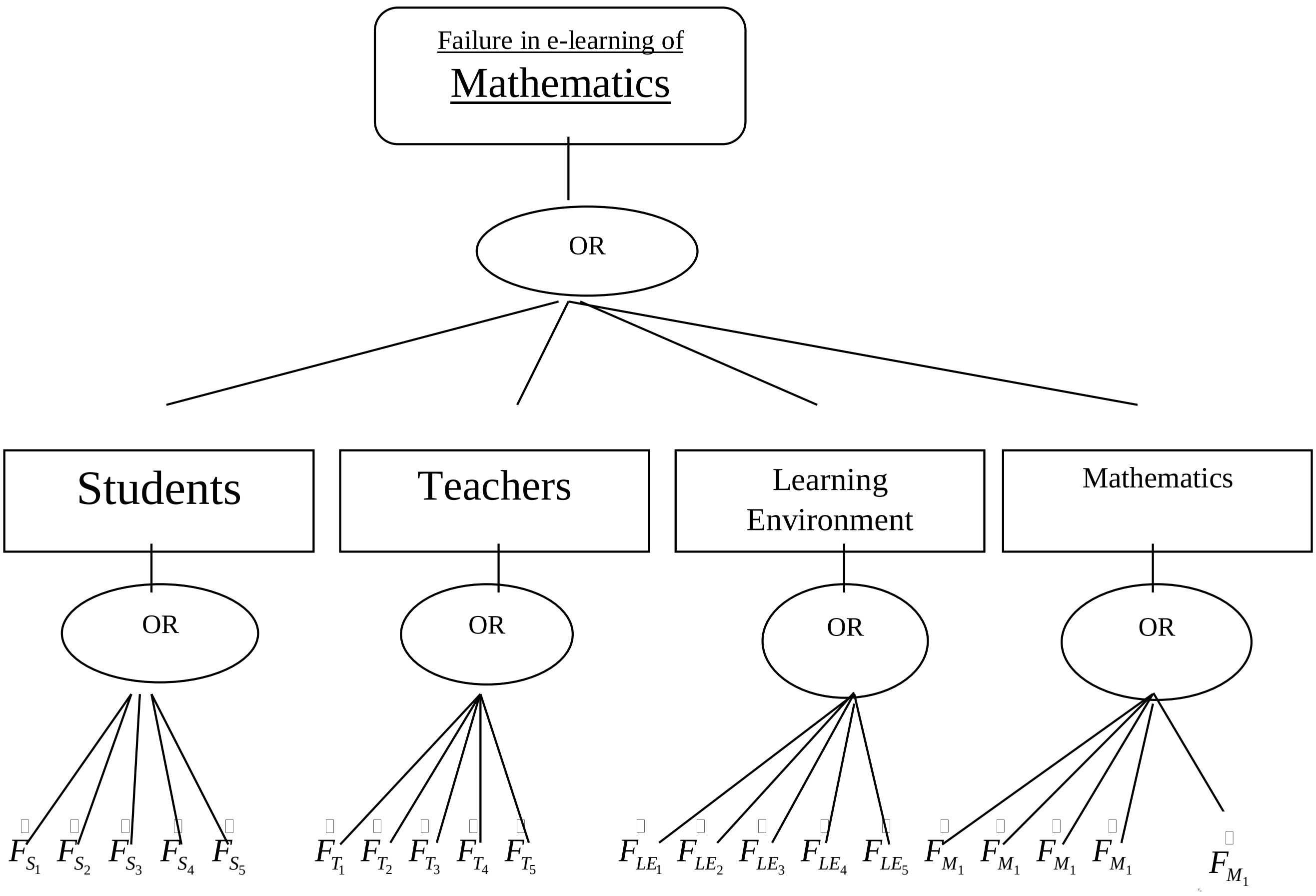

Figure 1 Fault Tree

The intuitionstic type-2 fuzzy barrier in e-learning of Mathematics can be calculated when the barrier of the occurrence of basic fault events are known. Barrier in e-learning of Mathematics of a truck can be evaluated by using the following steps.

Step1:

$$\tilde{F}_S = 1\Theta\left(1\Theta \tilde{F}_{S_1}\right)\left(1\Theta \tilde{F}_{S_2}\right)\left(1\Theta \tilde{F}_{S_3}\right)\left(1\Theta \tilde{F}_{S_4}\right)\left(1\Theta \tilde{F}_{S_5}\right)$$

$$\tilde{F}_T = 1\Theta\left(1\Theta \tilde{F}_{T_1}\right)\left(1\Theta \tilde{F}_{T_2}\right)\left(1\Theta \tilde{F}_{T_3}\right)\left(1\Theta \tilde{F}_{T_4}\right)\left(1\Theta \tilde{F}_{T_5}\right)$$

$$\tilde{F}_{LE} = 1\Theta\left(1\Theta \tilde{F}_{LE_1}\right)\left(1\Theta \tilde{F}_{LE_2}\right)\left(1\Theta \tilde{F}_{LE_3}\right)\left(1\Theta \tilde{F}_{LE_4}\right)\left(1\Theta \tilde{F}_{LE_5}\right) \quad \text{............................}\mathbf{(A)}$$

$$\tilde{F}_M = 1\Theta\left(1\Theta \tilde{F}_{M_1}\right)\left(1\Theta \tilde{F}_{M_2}\right)\left(1\Theta \tilde{F}_{M_3}\right)\left(1\Theta \tilde{F}_{M_4}\right)\left(1\Theta \tilde{F}_{M_5}\right)$$

Step2:

$$\tilde{F}_{e-L/M} = 1\Theta\left(1\Theta \tilde{F}_S\right)\left(1\Theta \tilde{F}_T\right)\left(1\Theta \tilde{F}_{LE}\right)\left(1\Theta \tilde{F}_M\right) \quad \text{...................................}\mathbf{(B)}$$

Result of Barrier in E-Learning of Mathematics Using TIT2FN

Numerical of barrier in e-learning of Mathematics using fault tree analysis with intuitionistic type-2 fuzzy failure rate. The components failure rates as TIT2FN are given by

$$\tilde{F}_{S_1} = (0.02, 0.03, 0.05; 0.2)(0.01, 0.03, 0.06; 0.2)$$

$$\tilde{F}_{S_2} = (0.01, 0.03, 0.04; 0.3)(0.001, 0.03, 0.05; 0.3)$$

$$\tilde{F}_{S_3} = (0.03, 0.04, 0.05; 0.1)(0.02, 0.04, 0.06; 0.1)$$

$$\tilde{F}_{S_4} = (0.02, 0.04, 0.06; 0.4)(0.01, 0.04, 0.07; 0.4)$$

$$\tilde{F}_{S_5} = (0.03, 0.05, 0.06; 0.3)(0.02, 0.05, 0.07; 0.3)$$

$$\tilde{F}_{T_1} = (0.02, 0.04, 0.06; 0.4)(0.01, 0.04, 0.07; 0.4)$$

$$\tilde{F}_{T_2} = (0.03, 0.04, 0.05; 0.3)(0.02, 0.04, 0.06; 0.3)$$

$$\tilde{F}_{T_3} = (0.04, 0.05, 0.07; 0.2)(0.02, 0.05, 0.08; 0.2)$$

$$\tilde{F}_{T_4} = (0.03, 0.05, 0.06; 0.1)(0.01, 0.05, 0.07; 0.1)$$

$$\tilde{F}_{T_5} = (0.03, 0.04, 0.07; 0.2)(0.02, 0.04, 0.08; 0.2)$$

$$\tilde{F}_{LE_1} = (0.02, 0.04, 0.05; 0.2)(0.01, 0.04, 0.06; 0.2)$$

$$\tilde{F}_{LE_2} = (0.03, 0.05, 0.06; 0.3)(0.02, 0.05, 0.07; 0.3)$$

$$\tilde{F}_{LE_3} = (0.04, 0.05, 0.06; 0.1)(0.03, 0.05, 0.07; 0.1)$$
$$\tilde{F}_{LE_4} = (0.03, 0.05, 0.06; 0.5)(0.02, 0.05, 0.07; 0.5)$$
$$\tilde{F}_{LE_5} = (0.02, 0.03, 0.05; 0.4)(0.01, 0.03, 0.06; 0.4)$$

$$\tilde{F}_{M_1} = (0.02, 0.03, 0.04; 0.4)(0.01, 0.03, 0.05; 0.4)$$
$$\tilde{F}_{M_2} = (0.03, 0.04, 0.05; 0.3)(0.02, 0.04, 0.06; 0.3)$$
$$\tilde{F}_{M_3} = (0.04, 0.05, 0.06; 0.5)(0.03, 0.05, 0.07; 0.5)$$
$$\tilde{F}_{M_4} = (0.02, 0.04, 0.06; 0.1)(0.01, 0.04, 0.07; 0.1)$$
$$\tilde{F}_{M_5} = (0.03, 0.05, 0.07; 0.3)(0.02, 0.05, 0.08; 0.3)$$

Using **(A)** in the step-1 we have the following results

$$\tilde{F}_S = (0.1053960436, 0.176223232, 0.23444896; 0.1)(0.05965324804, 0.176223232, 0.273985642; 0.1)$$
$$\tilde{F}_T = (0.1413572416, 0.20152576, 0.272985642; 0.1)(0.0775377208, 0.20152576, 0.3118717216; 0.1)$$
$$\tilde{F}_{LE} = (0.1325052544, 0.2016124, 0.25039794; 0.1)(0.0869506012, 0.2016124, 0.2892701548; 0.1)$$
$$\tilde{F}_M = (0.1325052544, 0.19320832, 0.250565824; 0.1)(0.0869506012, 0.19320832, 0.289432756; 0.1)$$

Using **(B)** in the second and final step, we get the barrier in e-learning of Mathematics, calculated fuzzy failure to start of a truck represented by the following TIT2FN,

$$\tilde{F}_{e-L/M} = (0.42193452489635460606240466663, 0.57631250737653777626303358689, 0.68733346225218227374509363332; 0.1)$$
$$(0.27685532129870893698465600065, 0.57631250737653777626303358689, 0.74769587488124274696887845811; 0.1)$$

CONCLUSION

Different technological tools are need for the math students. Such as computer algebra system, dynamic geometry environments, interactive applets, handheld computation, data collection and idea to different devices. This system gives a students to explore, identify math concept perfectly and also relationships. But fuzzy system only gives the idea of uncertainty or where the data are insufficient. But we have to learn it to know the reality, because it is an obstruction in the modern e-learning world. We use fuzzy data so present the barrier of e-learning of mathematics in reliability system. Our approaches and computational procedures may be efficient and simple to implement for calculation in an intuitionistic type-2 fuzzy environment for all fields of engineering and science where impreciseness occur. However in the future instead of taking component reliability as TIT2FN other more general fuzzy numbers like TrIT2FN may be used to evaluate system reliability in any branch of education.

REFERENCE

Alefeld,G.,and Herzberger,J. (1983). *Introduction to Interval Computation*. New York: Academic Press

Cheng.C.H. and Mon.D.L .(1993). Fuzzy System Reliability Analysis by Interval of Confidence", *Fuzzy Sets and Systems*, 56 (1993), 29-35.

Cai.K.Y ,Wen.C.Y. and Zhang. M.L. "Fuzzy reliability modelling of gracefully degradable computing systems", Reliability Engineering and System Safety, 33 (1991), 141-157.

Cai. K.Y., Wen. C.Y.and Zhang. M.L. (1991). Survival Index for CCNs: a Measure of Fuzzy Reliability Computing Systems", *Reliability Engineering and System Safety*, 33 (1991), 141-157.

Cai. K.Y. and Wen. C.Y. (1990). Streeting-Lighting Lamps Replacement: A Fuzzy Viewpoint", *Fuzzy Sets and System*, 37 (1990), 161-172.

Chakraborty, A.; Mondal, S.P.; Alam, S.; Ahmadian, A.; Senu, N.; De, D.; Salahshour, S. (2019). Disjunctive Representation of Triangular Bipolar Neutrosophic Numbers, De-Bipolarization Technique and Application in Multi-Criteria Decision-Making Problems. *Symmetry 2019, 11, 932*.

Chakraborty, A.; Mondal, S.P.; Alam, S.; Ahmadian, A.; Senu, N.; De, D.; Salahshour, S. (2019). The Pentagonal Fuzzy Number:Its Different Representations, Properties, Ranking, Defuzzification and Application in Game Problems. *Symmetry 2019, 11, 248.*

Chakraborty,A; Mondal,S.P.; Ahmadian,A.; Senu,N.; Alam,S.; Salahshour,S. (2018). Different Forms Of Triangular Neutrosophic Numbers, De.Neutrosophication Techniques, And Their Applications, Symmetry 10 (8), 327

Chen. S.M. and Jong. W.T. (1996). Analyzing Fuzzy System Reliability Using Interval Of Confidence", *International Journal of Information Management and Engineering*, 2(1996),16-23.

Chen, S.H. (1985). Operations On Fuzzy Numbers With Function Principle. *Tamkang Journal of Management Sciences*, 6(1), (1985), pp 13 – 26.

Dubois, D., and H. Prade, H. (1978). Operations of Fuzzy Number's, *Internat. J. Systems Sci.* 9(6) (1978), 613-626.

Dubois, D., and H. Prade, H. (1980). Fuzzy Sets And Systems, Theory And Applications. New York: Academic Press

Dwyer, P.S. (1951). *Linear Computation*. New York

Dwyer, P.S.. (1964). Matrix Inversion With The Square Root Method. Techno Matrices, 6(2)(1964).

Hansen, E.R. (1965). Interval Arithmetic in Matrix computations, Part *Journal of SIAM* series B, volume 2, number 2

Hansen, E.R., and Smith, R.R. (1967). Interval Arithmetic in Matrix computation Part II", *SIAM Journal of Numerical Analysis*, 4 1-9.

Hansen, E.R. (1969). On The Solutions Of Linear Algebraic Equations With Interval Coefficients", *Linear Algebra Appl.,* 2 153-165.

Hansen, E.R. (1992). *Global Optimization Using Interval Analysis*. New York: Marcel Dekker, Inc.

Hussain, S. A. I., Mandal, U. K., & Mondal, S. P. (2018). Decision Maker Priority Index And Degree Of Vagueness Coupled Decision Making Method: A Synergistic Approach. *International Journal of Fuzzy Systems,* 20(5), 1551-1566.

Kaufmann, A. (1975). *Introduction to theory of Fuzzy Subsets* Vol. I. New York: Academic Press,

Kaufmann, A., and Gupta, M.M. (1985). Introduction to Fuzzy Arithmetic. New York: Van Nostrand Reinhold

Lodwick, W.A., and Jamison, K.D. (1997). Interval Methods And Fuzzy Optimization. *International Journal of Uncertainty*, Fuzziness and Knowledge- Based Systems, 5 239-249.

Mahapatra. G.S. and Roy. T.K. (2009). Reliability Evaluation using triangular intuitionistic Fuzzy Numbers Arithemmetic Operations, *Proceedings of World Academy of Science, Engineering and Technology*, 38587-595

Majumder, P., Mondal, S. P., Bera, U. K., & Maiti, M. (2016). Application of Generalized Hukuhara Derivative Approach In An Economic Production Quantity Model With Partial Trade Credit Policy Under Fuzzy Environment. *Operations Research Perspectives*, 3, 77-91.

Moore, R.E. (1979). *Methods and Applications of Interval Analysis*. Philadelphia: SIAM,

Mon. D.L. and Cheng. C.H. (1994). Fuzzy System Reliability Analysis For Components With Different Membership Functions, *Fuzzy Sets and Systems*, 64 (1994),145-157.

Mondal, S. P. (2016). Differential Equation With Interval Valued Fuzzy Number And Its Applications. *International Journal of System Assurance Engineering and Management*, *7*(3), 370-386.

Mondal, S. P. (2018). Interval Valued Intuitionistic Fuzzy Number And Its Application In Differential Equation. *Journal of Intelligent & Fuzzy Systems*, 34(1), 677-687.

M. Deldago, J.L. Verdegay, M.A. Vila. (1989). A General Model for Fuzzy Linear Programming. *Fuzzy Set and System*. 29 (1989), 21-29.

Kar. R, Shaw, A.K. (2019). Some Arithmetic Operations On Triangular Fuzzy Numbers And Its Application In Solving Linear Programming Problem By Dual-Simplex Algorithm", *wjert, 2019, Vol. 5, Issue 6, 397-404.*

Kar. R, Shaw, A.K. (2018). Some Arithmetic Operations On Trapezoidal Fuzzy Numbers And Its Application In Solving Linear Programming Problem By Simplex Algorithm", *Intl. J. Bioinformatics and Biological Sci*.: (V. 6 n.2, p. 77-86)

Salahshour,S.; Ahmadian,A.; Mahata,A; Mondal,S.P.; Alam,S. The behavior of logistic equation with alley effect in fuzzy environment: fuzzy differential equation approach International ", *Journal of Applied and Computational Mathematics* 4 (2), 62.

Shaw, A.K. and Roy, T.K. (2015). Fuzzy Reliability Optimization based on Fuzzy Geometric Programming Method Using Different Operators, *The Journal of Fuzzy Mathematics* (USA) Vol.23, No.1, pp.79-88, 2015

Shaw, A.K. and Roy, T.K. (2015). Reliability Analysis of the System with Imprecise Constant Failure Rate of the Components", *IAPQR Transaction, Vol.40, No.1,* 2015

Shaw,A.K, and Roy, T.K. (2011). Generalized Trapezoidal Triangular Intuitionistic Fuzzy Number And Its Application On Reliability Evaluation, Fuzzy Number With Its Arithmetic Operations And Its Application In Fuzzy System Reliability

Analysis, *International Journal Of Pure Applied Science And Technology* , 5(2) (2011), 60-76.

Shaw,A.K, and Roy, T.K. (2012). Some Arithmetic Operations On Triangular Intuitionistic Fuzzy Number And Its Application On Reliability Evaluation, *International Journal of Fuzzy Mathematics and System (IJFMS)*, Vol.2, No. 4(2012), pp.363-382

S.C. Fang, C.F Hu, S.-Y. Wu, H.-F. Wang. (1999). Linear Programming with Fuzzy Coefficients in Constraint, *Computers and Mathematics with Applications*, 37 (1999), 63-76.

Zadeh, L.A. (1976). The Concept Of A Linguistic Variable And Its Applications To Approximate Reasoning – Parts I, II And III", *Inform. Sci*. 8(1975) 199-249; 81975 301-357; 9(1976) 43-80.

Zadeh, L.A. (1978). Fuzzy Sets As A Basis For A Theory Of Possibility, *Fuzzy Sets And Systems*, No.1, pp.3-28, 1978.