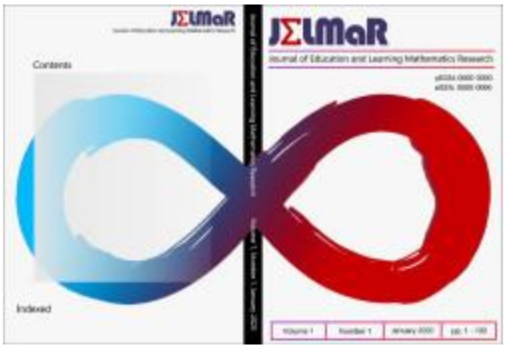

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The Identification of Students' Misconceptions in Mathematical Induction

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Abstract: *This study was conducted to identify the types of students' misconceptions, factors that cause students' misconceptions, and to describe the problem-solving of students' misconceptions in mathematical induction. The subjects of this study were 30 junior students of The Mathematics Department of Faculty of Teacher Training and Education of Universitas Islam Malang, who take The Real Analysis I course. This study employed a qualitative approach using a case study design. The study instruments used three tests and an interview guide. The data were analyzed using a qualitative approach in order to attain types of misconceptions and how to solve the misconceptions. With respect to the students' performance, the results of the study showed that most students had difficulty in interpreting letters, notation, generalization, and rules application. The factors that influence the students' algebra misconceptions in mathematical induction was the students' lack of ability, the lack of willingness that hinder the students' cognitive development, and the lack of learning motivation. In order to solve these problems, the researcher proposed a method called the guided discovery method.*

Keyword: misconceptions, mathematical induction

INTRODUCTION

Mathematics is a course that needs calculation in working out problems. There are many fields that need mathematics, such as economics, medical, engineering, etc.

Mathematics needs to be taught to young learners so that they are able to learn and master the knowledge earlier and easier. Mathematics needs to be taught based on learners' cognitive development. According to Piaget, 11-year learners will experience a formal operational level, in which learners will be able to think abstractly related to ideas and problem-solving alternatives (Huitt & Hummel, 2003).

Mathematics also needs to be taught to university students or pre-service teachers, who are considered as adult learners. The ability of university students must have developed perfectly to think of various alternatives to problem-solving. University students must be able to process information to be adapted and adopted. However, there are some students who are difficult to solve problems, especially in mathematics problems. Students' difficulties in learning mathematics are influenced by the complexity of materials, students' perception towards mathematics, and teachers' teaching strategy (Subanji, 2015).

Somehow, learning problems can distract the teaching and learning process. A misconception is one of the problems in which the concepts explained by teachers are not accepted perfectly by students. Maryati & Priatna (2018) states that misconception can be defined as a definition that is not accurate toward a concept, using the wrong concept, classifying the wrong concept, and the relation of incorrect concepts. Mathematics learning is cumulative; that is, new knowledge gained is linked to the previous knowledge. Hence, if a student is unable to "assimilate" and "accommodate" this creates a gap in the learning of the concept, and in turn, leads to mathematical errors or misconceptions (Roselizawati

Hj Sarwadi & Shahrill, 2014). Voutsina (2012) claimed that students conceptual knowledge is a source of information about the procedure. Many students still feel difficulties in understanding problems either the language or the context of the problems (Klymchuk, Zverkova, Gruenwald, & Sauerbier, 2010).

Suparno (2013) states that misconception can be caused by students, teachers or lecturers, learning contexts, teaching styles, and textbooks. In addition, misconceptions can also be caused by students' cognitive development which is not in line with the concept learned, students' reasoning limitations, students' ability in understanding concepts, and students' interests in learning concepts, so as in mathematics misconceptions. Mathematical misconceptions need more attention to solve, otherwise, they might affect the next mathematical problems (Subanji, 2015). Yensy (2018) also states that deep misconception analysis is needed so that the types of misconceptions and the causing factors can be identified earlier and we can find a solution for the problems.

Students' misconceptions can be detected through (a) giving well-structured assignments; (b) administering a diagnostic test in the beginning of the meeting; (c) providing direct questions to students; (d) providing flipped questions; (e) providing feedbacks; and (f) doing an interview (Maryati & Priatna, 2018). In fact, misconceptions in mathematics can also happen to students who take The Real Analysis course.

Real Analysis is a course that needs a deep understanding of concepts, in which concepts are the basic information to comprehend the material. If a student cannot understand the concept, the student cannot accept the material correctly and fully. One of the materials that are discussed in The Real Analysis course is mathematical induction. Important concepts that are used in mathematical induction are algebra and algebra manipulation concepts. Many students do miss algebra and its operational calculation during the mathematical induction process. As an effort to improve concept understanding ability, researchers consider this as an important thing to analyze students' misconceptions of mathematical induction concepts.

METHOD

To obtain data on students' misconceptions in mathematical induction, the researchers did a deep observation of the students' assignments. This study employed a qualitative approach using a case study design. In this study, the researchers observed and described the students' misconceptions and how to solve the misconceptions in mathematical induction. The subjects of the study were 30 junior students of The Mathematics Department of Faculty of Teacher Training and Education of Universitas Islam Malang, who take The Real Analysis I course.

In order to collect the data needed, the researchers used three tests and an interview guide. The data were in the forms of the students' works and the interview. The data were analyzed using a qualitative approach in order to attain types of misconceptions and how to solve the misconceptions. The researchers administered three tests to the 30 students step by step, beginning from basic algebra material, basic mathematical induction, and the last was a test of mathematical induction understanding. After administering the tests, the researchers did unstructured interviews with the three students to obtain further information related to students' misconceptions. Three research subjects were taken based on test scores who were considered to represent the overall abilities of students who experienced misconceptions.

RESULT AND DISCUSSION

The first test was used as a preliminary study. It aimed to identify the students who experienced basic algebra misconceptions. The second test was used to identify the types and the cause of the students' misconceptions in mathematical induction which involved algebra. After administering the second test, the researchers conducted an unstructured interview with the students to investigate the factors that cause misconceptions. The last test was used to obtain data of the students who did not experience mathematical induction misconception after they undertook some steps to overcome the misconceptions. The unstructured interview was also done after the researchers administered the third test. It was done to support that the students were not misunderstood the mathematical induction.

Based on test 1, which is related to basic algebra, it is obtained that 63.3% of the students misunderstood the concept of basic algebra, while 36.7% of the students did not experience the misconception. Based on the result of test 1, it shows that mostly the students misunderstood items number 1 and 4. The following figure is an example of misunderstanding item number 1 from a student.

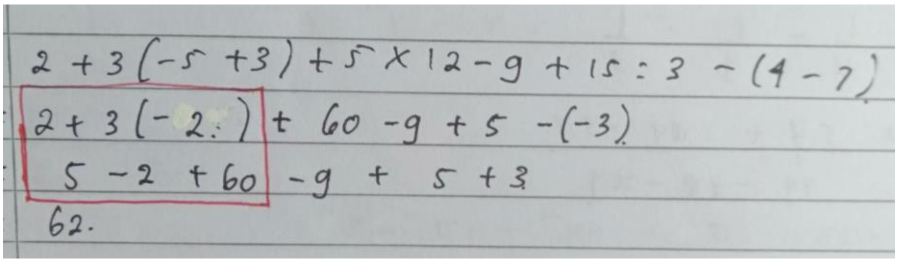

Figure 1. The Result of a Student's Work on Item Number 1 (Test I)

Considering the answer given by one of the students, it can be seen that the student did the basic algebra calculation incorrectly. In calculating $2 + 3(-2) = 5 - 2 = 3$, the student did the additional calculation first, which is placed at the beginning of the formula. Then the result of 2 + 3 is multiplied by (-2), so it is obtained 69 as the final result of the calculation. The student missed the existence of brackets ( ), in which the operation between brackets must come first because it is multiplication.

The following figure shows a misconception of item number 4.

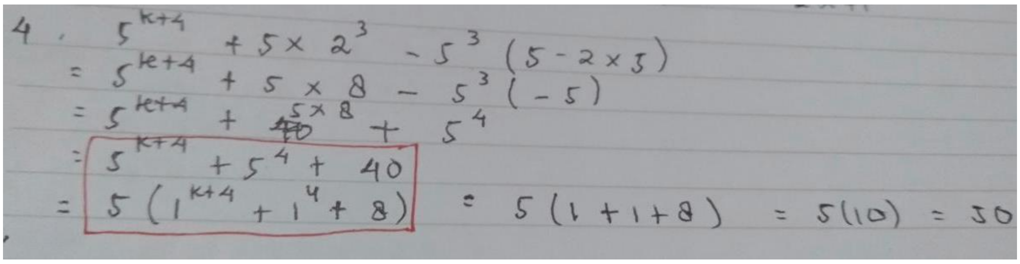

Figure 2. The Result of a Students' Work on Item Number 4 (Test I)

Considering the answer given by one of the students in figure 2, it can be seen that the student did the rank operation incorrectly. In calculating $5^{k+4} + 5^4 + 40 = 5\left(1^{k+4} + 1^4 + 40\right)$ the student did put number 5 outside the brackets, so the calculation becomes distributive.

Based on the first test, it shows that the student forgot about algebra basic concept. Students forget the order of performing operations in mathematics. Students also forget the importance of parentheses ( ) in mathematical arithmetic operations. Students also forgot about operations that occur in the rank. So that it makes many concept errors occur in the first test.

After administering the first test, the students were given mathematical induction. In this material, there are several patterns of numbers learned; they are series, division, and inequalities. After learning the materials, the students were given the second test to identify the types and the causes of misconception in mathematical induction.

Based on the second test, which is related to mathematical induction, it is obtained that 70% of the students misunderstood the concept, while 30% of the students did not experience the misconception. Based on the result of test II, it shows that mostly the students mistakenly carried out the rank operation. The following figures 3 and 4 are examples of misunderstanding item number 2 and 3 by students 1 and 2.

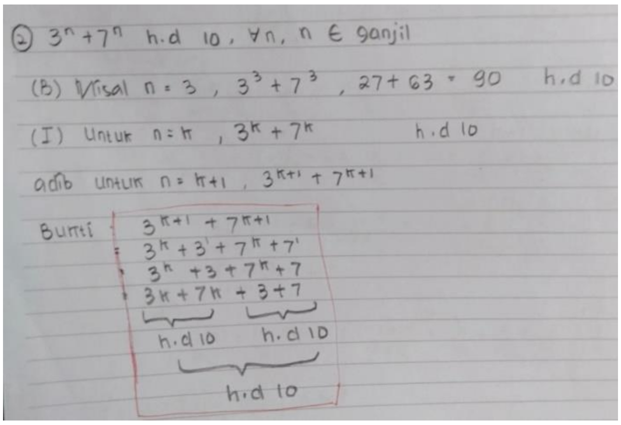

Figure 3. The Result of a Student's Work on Item Number 2 (Test II)

Based on the answer given by student 1 in figure 3, it can be seen that the student did the rank operation on addition incorrectly when he tried to calculate $n = k + 1$. In figure 3, it is written $3^{k+1} + 7^{k+1} = 3^k + 3 + 7^k + 7$. It shows that the student did not understand that $3^{k+1} = 3^k \times 3^1$. Since the student has mistakenly done the calculation from the beginning, the final result of the calculation is wrong.

Based on the second test, the results of the Analysis show that students forget about the operations that occur in the rank. This happened when students were doing simple math induction. When students do wrong operations on the rank, the process of proving mathematical induction will also be wrong. Thus the statement in the question becomes unproven.

Another misconception was found in item number 3 by student 2, which is about inequalities. The following figure is an example of work by the student.

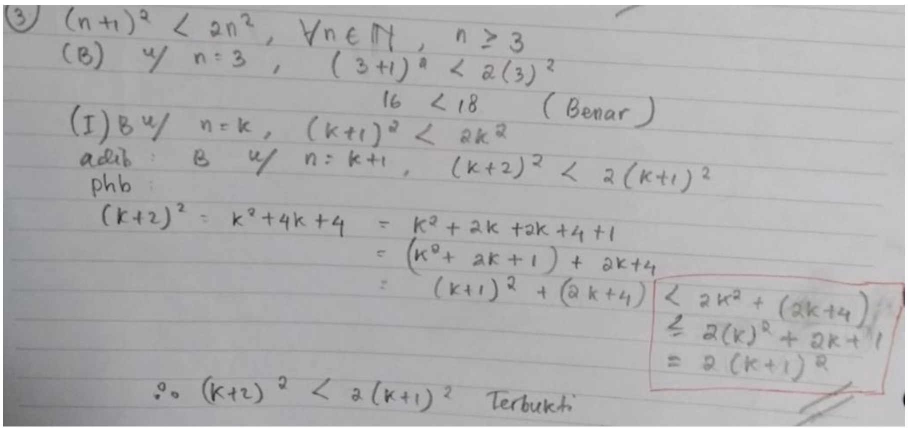

Figure 4. The Result of a Student's Work on Item Number 3 (Test II)

Based on figure 4, it can be seen that the student did the inequalities to prove $n = k + 1$incorrectly.

$$(k+1)^2 + (2k+4) < 2k^2 + (2k+4)$$
$$< 2(k)^2 + 2k + 1$$

$$= 2(k+1)^2.$$

The student wrote that $(k+1)^2 + (2k+4) < 2(k)^2 + (2k+1)$. Since it is written $n \geq 3$, therefore, $(k+1)^2 + 2k + 4 < 2(k)^2 + 2k + 1$ is incorrect. It is because $(2k+4) \nless ((2k+1)$, for $k \geq 3$. The subject's answer has a tendency to $2(k+1)^2$. So, the answer to $n = k+1$ is also wrong.

After the second test, the researcher conducted an unstructured interview with the subjects of the study. Based on the interview results, it is shown that the students' misconception in doing the tests because they forgot the rule of rank operation and were confused about determining steps to answer the questions.

In item number 2, test 1, the students mistakenly calculated the addition, in which the students thought that the rank addition means that they can add each rank. Therefore, it can be said that the students misunderstood the notation. In item number 3, test II, the students misapplied the rule.

After the researcher validated the data, it can be seen that the students' misconceptions were caused by the low ability to memorize, both in the sequence of arithmetic operations and also remembering concepts, and the students' cognitive competence do not develop as the learning materials given by the lecturer. The students work on mathematical induction tests, which were included in algebra proofs. The students could not understand the concept of algebra, and the results of the interview showed that the students forgot how to carry out the tests correctly. In fact, when the students said 'forgot', it can be related to their incapability to retrieve information of algebra (Stenberg, 2009:246). It is called memory distortion, in which it happened in seven ways, which (Schacter, 1999)called it "seven sins of memory". Two out of seven sins are:

a. *Transience*. People forget their memory quickly, in which they forget information right after they get it.
b. *Absent-mindedness*. Memory malfunctions due to distraction. It often occurs because an individual focuses on issues or concerns that end up distracting us, so the brain cannot code information safely (Stenberg, 2009).

These two aspects occur to the students, so they cause the misconceptions and hinder them in solving mathematical induction problems.

After knowing the cause of the students' misconceptions in mathematical induction, the researchers tried to solve the problems based on the causes. Hence, the researchers employed a guided discovery method to overcome the problems. The learning is not only finding a brand new knowledge but also it is expected that the students will find active knowledge through finding concepts, formulas, etc. The students are expected to find it under the lecturer's supervision because most of the students still need basic concepts (Amiyani & Widjajanti, 2018). Further, Amiyani & Widjajanti (2018) said that the guided discovery method is appropriate to be implemented in mathematics because it has may formulas and concepts. Therefore, the lecturers can plan a learning activity in which the students can associate concepts to find a new concept. By using the guided discovery method, the students can find their own concept in carrying out mathematical induction. In addition, they can use various ways to answer questions. In other words, the lecturer can facilitate the students to develop their cognitive readiness and capability.

After implementing the guided discovery method, the third test was conducted. This test was done to investigate whether the students did not misunderstand the concept of mathematical induction. The following is the answer to item number 1b.

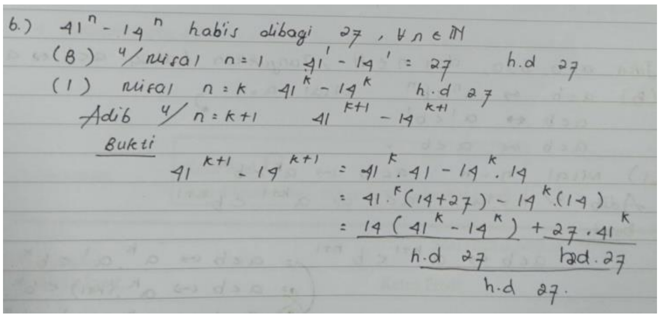

Figure 5. The Result of a Student's Work on Item Number 1b (Test III)

Based on the student's work after experiencing the guided discovery method, the student can prove the mathematical induction correctly. The following figure 6 shows the student's answer for item number 3.

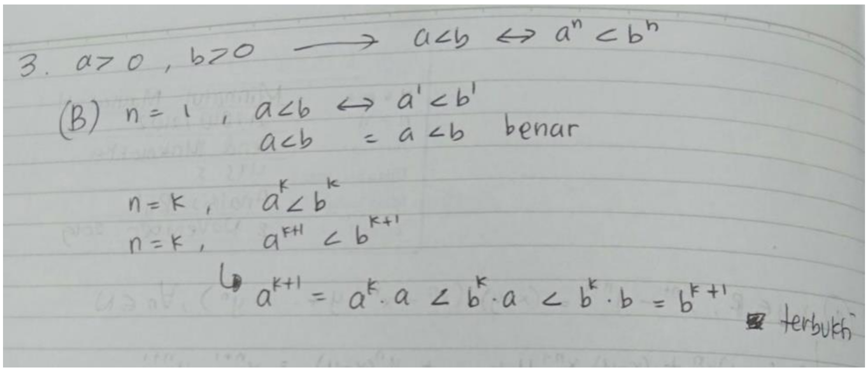

Figure 6. The Result of a Student's Work on Item Number 3 (Tes III)

Based on the students' answers, it can be seen that the students are able to prove the mathematical induction correctly and appropriately. It means that the students understand the concepts given through the implementation of guided discovery in mathematical induction.

An unstructured interview was also done after the third test. Interviews were conducted with three students based on test scores who were considered to represent the overall abilities of students who experienced. It aimed to reassure that the students understood the concepts of mathematical induction. Based on the results of the interview, it can be assumed that the students comprehend the basic concepts to solve mathematical induction, which includes the concept of algebra. Moreover, the students also can prove the mathematical induction problems in other ways.

CONCLUSION

Mathematical induction is a mathematical work that involves natural numbers. The steps to prove the mathematical induction problems have a particular procedure. Based on the results of the study, the misconceptions faced by the students of The Mathematics Department of Universitas Islam Malang in mathematical induction is related to the misconception of interpreting letter, notation, generalization, and rule application.

The factors that cause the students' misconceptions of algebra are the low ability of students to memorize, the common understanding of students' mathematical concepts which causes students not to focus on solving math induction problems. In order to solve these problems, the researcher proposed a method called the guided discovery method. By using the guided discovery method, the students can find their own concept in carrying out mathematical induction. In addition, they can use various ways to answer questions.

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